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The need for a new national policy and program in secondary mathematics[1]

WILLIAM DAVID REEVE, *Professor Emeritus of Mathematics, Teachers College, Columbia University, New York, New York.*

A forceful plea for action on the present "chaotic conditions" in mathematics from one who has had a long and rich experience with the mathematics program in the high school.

RECENT COOPERATIVE REPORTS ON SECONDARY MATHEMATICS

For some time now, I have been making a careful study of reports issued recently throughout the country by various cooperating groups on what ought to constitute the program in mathematics for the secondary school. The following selected list of samples is typical:

1. *Functional Mathematics in the Secondary Schools.* Bulletin No. 36, State Department of Education, Thomas D. Bailey, Superintendent, Tallahassee, Fla., June, 1950. Price, $1.
2. *General Mathematics in the High School.* Mathematics Bulletin No. 2, Wisconsin Cooperative Educational Planning Program, Room 147N, State Capitol, Madison, Wis., June, 1950.
3. *Education for the Talented in Mathematics.* Bulletin No. 15, U. S. Department of Health, Education and Welfare, Washington, D. C., 1952. Price, 15¢.
4. "Mathematical Needs of Prospective Students at the College of Engineering of the University of Illinois." Urbana, Ill., April, 1952.
5. *A Course of Study in Mathematics for Secondary Schools.* Bulletin No. 360, Department of Public Instruction, Harrisburg, Pa., 1952. Price, $1.25.
6. *An Experimental High School Mathematics Program.* A report presented to the ASEE meeting in Florida by Professor D. S. Babb, Department of Electrical Engineering, University of Illinois, Urbana, Ill., 1953.
7. *A Guide for Instruction in Mathematics.* Secondary School, Grades 7–12. Curriculum Bulletin, No. 20, State of Minnesota Department of Education, St. Paul, 1953. This bulletin can be had from Syndicate Press, 501 Park Avenue, Minneapolis, Minn. Price, $1.00. The program for grades 7–10 is excellent. The eleventh and twelfth grade programs need further thought in the light of recent developments.
8. *Developing Mathematical Literacy in Nebraska's Youth.* Prepared under the direction of Dr. Milton W. Beckmann, Supervisor of Mathematics, Teachers College, University of Nebraska, Lincoln, Neb., 1953.
9. *Mathematics for All High School Youth.* Report of Basic Skills Conference, Clinics in Mathematics, Bureau of Secondary Curriculum Development, State Education Department, Albany, N. Y., 1953.
10. *Mathematics in Public High Schools.* Bulletin No. 5, 1953, U. S. Department of Health, Education, and Welfare, Office of Education, Washington, D. C. Price, 20¢.
11. "Report to the Illinois Section of the Mathematical Association of America of Its Committee on the Strengthening of Mathematics Teaching." Presented to the Association at the Navy Pier, Chicago, Illinois, May 9, 1953. *The American Mathematical Monthly,* November, 1953, pp. 652–661.

    This is the report referred to by the former president of the National Council, John R. Mayor, on pages 491–494 of the November 1953 issue of THE MATHEMATICS TEACHER. Every teacher of mathematics should read it.
12. *College Admission with Advanced Standing.* Announcement and Bulletin of Information published by The School and College Study of Admission with Advanced Standing, January 1954. Executive Director, William H.

[1] A paper read at the Fourth Annual Meeting of The New York State Mathematics Teachers Association, Syracuse, N. Y., on Saturday, May 8, 1954; at the Sixth New England Mathematics Teachers Institute at the Massachusetts Institute of Technology on August 16, 1954; and also at the regular monthly meeting of Section 10 (Mathematics) of the New York Society for the Experimental Study of Education at Columbia University, on October 16, 1954.

Cornog, President, Central High School, Philadelphia 41, Pa. See also Brinkmann, H. W. "Mathematics in the Secondary Schools for the Exceptional Student," *The American Mathematical Monthly,* May, 1954, pp. 319–323.

13. *General Education in School and College.* A Committee Report by Members of the Faculties of Andover, Exeter, Lawrenceville, Harvard, Princeton, and Yale. Harvard University Press, Cambridge, Mass., 1952. Price, $2.00. See also "Mathematics in School and College," *The American Mathematical Monthly,* June–July, 1953, pp. 380–383.
14. *Mathematics 10-11-12. An integrated sequence for the senior high school grades.* Bureau of Secondary Curriculum Development, State Education Department, Albany, N. Y., 1954. One of the best syllabi yet published for the years concerned. The tentative junior high school syllabus (grades 7–8–9) is just as good, but has not yet been released.
15. *Mathematics in Secondary Schools Today.* Bulletin of the National Association of Secondary School Principals, Bulletin No. 203, May, 1954. Copies may be obtained for 75¢ each plus postage from Paul E. Elicker, 1201 Sixteenth Street, N.W., Washington 6, D. C.
16. *A Look Ahead in Secondary Education.* Report of the Second Commission on Life Adjustment for Youth, Bulletin No. 4, 1954, U. S. Department of Health, Education and Welfare, Office of Education, Washington 25, D. C. Price, 35¢.
17. *Teaching Rapid and Slow Learners in High Schools.* Bulletin No. 5, 1954, U. S. Department of Health, Education and Welfare, Office of Education, Washington 25, D. C. Price, 35¢.

I have no doubt that, in most of the above reports, if not in all, the groups that made them honestly tried to solve the most perplexing problem of the secondary school, namely, the problem of providing satisfactory courses to meet the needs of the different groups of students of varying ability who feel that they must study mathematics. However, I am compelled to say that, in general, most of the above reports are disappointing, to say the least. Much of the work is mediocre, repetitious, and even chaotic in spots. In some of the reports, the people making them do not seem to know what they wish to do, or how it ought to be done. Many of the recommendations made *could not be carried out* and, in some cases, many teachers of mathematics, unless they already know what to do, *would not benefit from anything that they read in some of these reports.*

In my opinion, it would have been better if most of the groups concerned had merely approved the Report of the Joint Commission of The Mathematical Association of America and The National Council of Teachers of Mathematics on *The Place of Mathematics in Secondary Education*[2] with whatever revisions they felt were necessary to bring the report up to date. In fact the Joint Report is still better than any of the more recent ones.

Many teachers will not know how to implement what they are requested to do in the conflicting reports that have been issued recently. Some of the groups reporting admit that only well-qualified teachers should be put in charge of the courses they recommend. Does this imply that just anyone can teach algebra, geometry, and the like? Some school superintendents and principals act as if they think so and this causes no end of confusion and trouble.

Some of the reports, as I shall point out later, have made good recommendations for two, or even three grades, but I have yet to find a complete secondary program that I think would be generally acceptable throughout the country, although the New York State syllabi come closest to being satisfactory.

It is only fair to say that there is one big advantage in a state report, provided it is a good one, and that is that the state usually makes it possible for each teacher, as is true of the Wisconsin report, to have a copy. However, there is no great value in getting reports out if they are not carefully read and studied by the teachers. It would be interesting to know how many teachers in this country have never heard about, much less read, the two or three outstanding national reports in recent years.

[2] *The Place of Mathematics in Secondary Education.* The Fifteenth Yearbook of The National Council of Teachers of Mathematics, The National Council of Teachers of Mathematics, 1201 Sixteenth Street, Washington 6, D. C., 1940. Price, $3.00.

WEAKNESSES OF SOME OF THE RECENT STATE REPORTS

Now what are the weaknesses of some of the recent state reports that should be pointed out for all teachers to see?

1. No report is of any value that does not give a genuine foundation for the mathematical structure, namely *the arithmetic that is fundamental in the elementary school.* It is not possible to develop a modern and worth-while curriculum in mathematics by digging in at some point, say the ninth grade, and then working upward from there. I have seen the sad results of this method many times. We must have a good foundation for the secondary school program, beginning in the seventh grade and ending with the twelfth.

2. Several of the reports show the results of very little thinking. The groups that made them are merely aping some previous report as, for example, the reports of the Commission on Post-War Plans in Mathematics which submitted a check list of twenty-nine competencies which is supposed to solve the problem. This is ridiculous. There is nothing sacred, permanent, or magical about this check list of twenty-nine items. Such groups act as if this list of competencies is satisfactory, when, as a matter of fact, it gives little help except for the ninth grade, where those who favor some kind of composite and terminal type of "consumer mathematics" course think it meets most of our present needs. Any teacher who is worth his salt knows that the twenty-nine competencies in the above report, as good as they may be, *cannot be covered* in one year and are by no means adequate to meet the needs of the average American citizen, no matter what his future may be. The Commission had only twenty-eight competencies and then added the twenty-ninth when it was pointed out that there was no reference to the student's learning anything about the meaning of proof.

3. In some of the new reports, many different terms are loosely used without any clear attempt to show what they mean. For example, I have yet to find a clear statement as to what is meant by "general mathematics"; where the term is used it almost always refers to material that is neither "general" nor "mathematics" in the real sense—it is arithmetic, or what is too frequently referred to as "consumer mathematics." The term "producer mathematics" and others receive similar treatment. What is the poor uninitiated teacher expected to do? Even in current meetings of mathematics teachers we hear a lot of discussion on "general mathematics" but seldom does anyone define what he means by the term.

One reason for this situation is the fact that we have not been able to develop a basic course in *real* general mathematics, as I shall define the term later in this paper, and, as a result, a type of mathematics has grown up Topsy-like which is intended for the slow-learning students who their teachers think are not able to do the sequential or college entrance type of course. Those of us who write textbooks are partly to blame, but the best we have been able to get publishers to do is to produce a two- or three-book series for either the junior or senior high school separately. What we need is a six-year series, but, although the trend now seems to be that way if we can judge by some of the new books, as yet there is none. Certainly the new New York State syllabi indicate that teachers favor a departure from the watertight compartment type of traditional course that has been the pabulum of generations of students.

For the reasons stated above, and because of certain bad influences on the students, the "consumer mathematics" type of course is certainly no substitute for a course in general mathematics in the secondary school.

4. There is no general agreement in these reports as to where differentiation in instructing high school students should begin, or how long it should continue. The expressions "first track" and "second

track" are bandied about by most of the groups without caution as to the possible stigmatizing effect upon any student who, for one reason or another, finds himself in a track other than the first because "first track" connotes "superior track." While conceding some value to a "second track" in their report, the Illinois group (the Babb report) comes out strongly for "one track."

In discussing the "two track" idea they said: "The mathematics programs given in high schools are usually of either of two types. Those that can have only one program will give their students the traditional courses in mathematics—one year of algebra and one year of plane geometry.

"Many of the larger high schools, which have the facilities and teacher personnel to do so, have adopted a 'two-track' mathematics program consisting of the traditional program and a 'general mathematics' program."[3] *The general mathematics programs are usually geared to the slower students, leaving the better students unchallenged. Textbooks for these general mathematics courses are usually written for those students with more general interests.* This situation arises from the desire to satisfy the needs of a large majority of the students. At the same time, *these general mathematics courses are based on the belief, which we vigorously oppose, that the general needs of all students do not go beyond simple arithmetic skills, applications of these skills, simple formulas, and some intuitive geometry.* We agree that the mathematical needs of all students *must* be considered, but extensive and cooperative studies are essential in order to determine these needs and then to find an effective procedure for teaching those essentials. All skills and manipulations should be based upon insight into fundamental mathematical concepts. It is our firm conviction that many current prejudices against mathematics arise from a false concept of mathematics as a study of meaningless manipulations. These false prejudices must be removed. They can be overcome if we teach the significance of mathematical operations and their basis upon fundamental but inherently simple mathematical concepts as we develop mathematical skills.

"A 'two-track' mathematics program has some merits. It also has the disadvantage of requiring the student to make an early decision as to his or her life work. Too often the choice between the college preparatory and the general mathematics is made on a superficial basis. *Indeed, there does not appear to exist a scientific basis for such a decision.* As soon as a start has been made in either one of the two programs, the student usually finds it difficult to transfer to the other program because of scheduling problems, serious duplication of subject matter, or lack of mathematical background.

"Most secondary schools appear to feel that, with the possible exception of remedial work, the college preparatory and the 'two-track' programs are the only two alternatives. We feel that our experimental program (an integrated mathematics program) represents a third alternative."[4]

The Babb report then goes on to explain what the Illinois group intends to do and gives a list of five assumptions upon which their program in mathematics will be based.[5]

In a few instances, groups of cooperating leaders are trying to stretch the one-year course into at least two years so as to be able to cover the check list of twenty-nine items of competency recommended by the Commission on Post-War Plans in Mathematics. As a result, there is beginning to appear, in such groups, a feeling that the one-year course is no longer satis-

[3] I think the Babb report is here referring to the usual "consumer mathematics" type of course which is not what I mean by "general mathematics."

[4] Babb report, pp. 6–7. The italics are mine.

[5] *Ibid.*, pp. 7–9.

It will be interesting to see what kind of a "one-track" program the Illinois group will set up. Whether one agrees with their tentative plans or not, it is clear that they think that the present situation is not satisfactory.

factory for any group of students in the secondary school. If the course is to be only for one year, the trend seems to be to offer it in the twelfth year.

There is such widespread disagreement between some of the reports referred to above, and such lack of unity in general, that some new and better plan of recommending a program for the secondary school seems advisable for the nation as a whole.[6]

5. In attempting to sugar-coat the course in mathematics for the slow-learning students, we are, at the same time, lowering the standards of accomplishment for the more gifted and, as a result, the bright student becomes the most retarded of all.[7] Some teachers seem to be interested mainly in caring for the dull. We should try to "level up" not "level down" in arranging the course in mathematics, so that ultimately we may gain instead of lose in our attempt to improve mathematical instruction. Even in the larger towns and cities, where homogeneous classification is possible, or where they have "two tracks" in order to try to solve the problem of varying abilities, the teachers say that the slower learning students refuse to elect the "second track" because of the stigma that is attached to it. Moreover, the parents support their children in this attitude.

At a recent meeting of mathematics teachers, I heard a supervisor in one of the largest American cities say that they were solving the discipline problem in their schools by putting the gifted students in the same class with the slower ones. This would seem to me to be a good way to create discipline problems if I know anything about gifted children. So the gifted student loses his chance to develop himself to the fullest in more ways than one, and the slow-learning student becomes discouraged and gives up the ghost.

If we are to avoid all of the accompanying embarrassment of such situations of those described above, we shall have to formulate, or at least implement, a better guidance program in the schools.

THE REAL NATURE OF GENERAL MATHEMATICS

What we need to do is to clarify the minds of classroom teachers as to the merits of the genuine general mathematics program over momentary interests as the easiest way out of our difficulty. The fact is that many students who are advised to take consumer mathematics (general mathematics as they call it), or who, for one reason or another, may be forced to take it, are really being handicapped for life's further pursuits.

General mathematics, as I define it, *is a completely reorganized course in informal geometry, arithmetic, algebra, formal geometry, trigonometry (numerical at first), an introduction to analytical geometry and the calculus, beginning in the seventh grade and running continuously through the secondary school, and in such a way as to show how these various subjects are related in order that they may reinforce and supplement each other in useful ways.*[8]

When the junior high school syllabus is published, New York State will have a very good six-year program in mathematics for secondary school except for the twelfth year which needs further revision. For example, both the Minnesota and New York State courses of study give a half year to solid geometry, which can no longer be justified. Of course most secondary schools do not teach solid geometry anyway, but many of them do, and a great

[6] See the Nebraska report, *op. cit.*, pp. 3–4.
Here again this report evidently considers general mathematics to be a consumer mathematics type of course.

[7] See W. D., Reeve, *Mathematics for the Secondary School—Its Content and Methods of Teaching and Learning* (New York: Henry Holt and Company, 1954), pp. 56–64.

[8] For a more complete discussion of the history, nature, and purpose of general mathematics in the secondary school, see W. D. Reeve, "General Mathematics in the Secondary School," February and March issues of THE MATHEMATICS TEACHER, 1954; or W. D. Reeve, *Mathematics for the Secondary School—Its Content and Methods of Teaching and Learning*, *op. cit.*, Chapter 11.

deal of time is lost that might be better spent on other topics. The thirteenth report in my list made the following point about solid geometry in secondary school:

"The impact of these notions upon the present mathematical curriculum is heavy, for large parts of it turn out to be relatively unhelpful elaborations of principles which are better taught in other ways. The greatest single offender in this sense is solid geometry. It is a beautiful subject, but in the strictly mathematical sense it is an elaboration of plane geometry, and elaboration is not the point of mathematics. The real value of solid geometry lies outside its mathematics, in the fact that it tends to develop a general sense of spatial reality. This can be done more briefly and more effectively, we think, if the effort to develop a systematic structure is abandoned. If generally adopted, this single revision would save nearly half a year in the standard school curriculum."[9]

The report goes on to say that certain other topics like "complex numbers, determinants, logarithmic solution of triangles and the geometry of the circle . . . seem appropriate for condensation or omission."[10]

The report concludes with a strong plea that more attention be paid to "the calculus and to statistics." It said:

"To some of our consultants, and in some respects to our Committee as well, the case for statistics is even more powerful than the case for the calculus. The notions of probability, correlation, and sampling are among the fundamentals of modern social measurement. And since we live in the age of polls, an awareness of the real meaning of these notions is a protection to the consumer as well as a necessity for the producer of information. Moreover, there is in all statistics a salutary concern for the uncertain and the incomplete—for the gray that is real more than for the black and white that is abstraction. It is well for the student to learn both that mathematics has uncertainty and that uncertainty can be mathematically treated. This knowledge is important in many fields; teachers of science and teachers of history alike have their troubles with students who are persuaded that all reasoning is geometrical and all evidence conclusive. All in all, if we had a curriculum to build from the ground up, we cannot suppose that it would omit statistics from a general education.

". . . Finally, nearly all good school departments of mathematics are well equipped to teach the calculus; they have been teaching it, on a limited scale, for many years. On balance, therefore, we recommend that the schools should move toward a curriculum in which the basic 12th grade course is the introduction to the calculus. At the same time we hope that there will be intensive experimentation with the teaching of some of the basic concepts of statistics, and we think there is room for this in the second year's study of algebra. Nor do we exclude the possibility that some schools may wish to offer statistics as an alternative to the calculus, or even as an additional elective.

"It is our conclusion that the school mathematics curriculum can and should be redesigned to include new areas of instruction; and we think that when this has been done, college mathematics should be a subject for scientists, mathematicians, and really talented amateurs of the topic. For others, there is plenty in the basic course we have outlined, and their college work should be in other fields. And this we feel is as it should be; fundamental mathematics of the sort we have been discussing is taught better—and learned better—in the schools than in the colleges."[11]

[9] *General Education in School and College*, p. 54. See also "Mathematics in School and College," *The American Mathematical Monthly*, *op. cit.*, p. 381.

[10] *General Education in School and College*, p. 54.

[11] *Ibid.*, pp. 56–57. See also W. D., Reeve, *Mathematics for the Secondary School*, Chapter 10, which gives a full discussion of "Mathematics for the Citizen."

Whether one agrees with any or all of the preceding reports, the fact remains that they do not agree with each other, which makes the need for a new national program in secondary mathematics more necessary than ever.

Betz, in a recent report to Section 7 of the International Congress of Mathematics, at Cambridge, Massachusetts, September 1, 1950, said:

"Three principal approaches have been tried or suggested in American secondary schools in their attempts to provide functioning types of mathematical training for *all* American youth.

"The *first* is anchored on the popular doctrine that 'life situations' should be the primary basis of all curricula. But since mathematics is a *system* of ideas and processes, whereas life situations are incurably *unsystematic*, this approach has failed completely wherever it has been tried. It has always resulted merely in a sort of chaotic 'mathematics without mathematics,' and it has ignored fundamental aspects of the problem we are considering.

"The *second* approach hopes to find dependable answers in the recommendations of authoritative committees and in the techniques of curriculum workshops and laboratories. However, the thousands of mathematical syllabi now crowding the shelves of our curriculum morgues have merely dramatized a hopeless confusion of objectives. And even the reports of *national* committees have regularly been attacked by leading educators as mere reflections of unsupported private opinion.

"There remains a *third* approach, often explored partially, but never with anything like scientific completeness or thoroughness. It is that of making a really dependable, full-length study of the role of mathematics in the modern world, from both a practical and a cultural standpoint."[12]

This report was followed by a set of resolutions which were adopted unanimously, and recommended to The International Mathematical Union which subsequently adopted them.

THE CORE CURRICULUM

In many secondary schools there is a movement to modify the compartmentalization of subject matter and organize the students under faculty chairmen or homeroom teachers throughout the school day. Such teachers are chosen for their broad experience and capacities. They coordinate plans for the groups under their care, calling in the mathematics teacher, the science teacher, and others, somewhat as music and physical education teachers are called in at special intervals to work with children in the elementary grades. Frequently the "specialist" is asked to contribute toward the development of some central theme, such as "Consumer-Producer Relationships." This kind of organization is sometimes designated as the Core Curriculum.[13] In view of the changing character of public secondary schools, the teacher should make it his business to study such procedures with care, whenever he has the opportunity. By doing so, he is

[12] William Betz, "Mathematics for the Million, or for the Few?" THE MATHEMATICS TEACHER (January 1951), 44: 20.

[13] Fawcett, H. P. "Mathematics and the Core Curriculum," The Bulletin of The National Association of Secondary-School Principals, *Mathematics in Secondary Schools Today, op. cit.*, 71–80. See also Edwards, Karl D., "Meeting the Needs of Youth Through a Core Program," *The Bulletin of Education*, University of Kansas, February 1954; Jansen, H. S., "The Relation of Mathematics to the Core Curriculum," THE MATHEMATICS TEACHER, October 1952, 45: 427–435; Syer, Henry F., "A Core Curriculum for the Training of Teachers of Secondary Mathematics," THE MATHEMATICS TEACHER, January 1948, 41: 8–21; Fehr, Howard, "Socializing Mathematics Instruction," THE MATHEMATICS TEACHER, January 1948, 41: 7; Alberty, Harold, *Reorganization of the High-School Curriculum*, The Macmillan Company, New York, 1947, pp. 154–155; Mannheimer, W. P., "Mathematics in the Core Curriculum," *High-points*, New York Board of Education, October 1944, 26: 71–73; Adler, Ruth and Peters, Max, "General Mathematics and the Core Curriculum," THE MATHEMATICS TEACHER, March 1953, 46: 171–177; *Core Curriculum Development and Problems*, Bulletin No. 5, Department of Health, Education and Welfare, Office of Education, Washington, D. C., 1952; Tyler, Ralph W., "The Core Curriculum," *N.E.A. Journal*, December 1953, pp. 563–565.

led to see how broad his preparation must be if he is to measure up to the demands of teaching, either as a core chairman, or as a mathematics specialist, in a system organized along such lines.

If the core curriculum idea is to have any chance of ultimate success, plans must be made to make teachers generally more scholarly. No one would suggest that one teacher can correlate all of the various subject-matter fields and teach them in the classroom. But it is both possible and desirable to have the teacher of mathematics teach science, particularly some physics or general science.

In report number 11 of my list we find the following statement:

"The Committee wishes to go on record as favoring the subject-matter type of curriculum as opposed to the core type of program, specific subject-matter requirements for high-school graduation and for college entrance and mathematics for all those able to comprehend it."[14]

THE TYPE OF ACTION NEEDED NOW

One of the most important things for us to keep in mind is the fact that we need more and better cooperation between the college professors of mathematics and the teachers of mathematics in the secondary school. We have made some progress, but much more needs to be accomplished.

What is definitely needed now is a national policy group to produce a good course of study in mathematics, and thus give prestige and respectability to the course. Besides, such a national effort would avoid the waste involved when every Tom, Dick, and Harry in the country tries to set up a course of study suited to his particular whims. We have had such national reports in the past, but the trouble is that even the best reports have not been studied universally and they have not been widely supported by the general educators and school administrators.

If the Board of Directors of The National Council of Teachers of Mathematics were only interested enough, they could organize a national commission composed of a few of the best available people from the elementary, secondary, and collegiate fields that could set up some kind of policy and program which would be adequate to meet our needs. This policy and program could then be referred back to local groups for discussion and approval. After a few years, we could, in this way, develop recommendations that would be trustworthy and helpful. It might even involve some rather careful experimentation in the classes of teachers who are qualified for such work. We might well decide, for example, whether the formal part of the instruction in solid geometry and trigonometry could not be completed by the end of the eleventh grade so as to leave the twelfth grade open for the calculus or other alternative courses. Here again, we are stymied because, although some of us have pleaded year after year for such action, our efforts have been in vain.

If we cannot induce some national organization like The National Council of Teachers of Mathematics to lead the way, then, I think, we should work cooperatively, secondary and college teachers alike, to try to set up a national institute for mathematical education, strategically located, that would help secondary teachers of mathematics to do better what they are going to do anyway after some fashion or other. We could do this if we all work together to obtain financial aid.

[14] "Report to the Illinois Section of the Mathematical Association of America of Its Committee on the Strengthening of Mathematics Teaching," p. 658.

What does "if" mean?

KENNETH O. MAY, *Carleton College, Northfield, Minnesota.*

Words can be slippery customers in mathematics as well as in other fields. The only way to keep them from tripping the students (and the teachers) is to really understand their use. Here the use of "if" is clarified.

THE RELATION OF IMPLICATION, expressed by sentences of the form "If H, then C," "H implies C," or "Hyp: H, Con: C," is so basic in mathematics at every level, and in all logical thinking, that a thorough understanding of it ought to be a major goal of teaching. Nevertheless, there is an amazing amount of confusion about it. A very large part of the difficulties that students experience in all subjects, and especially in advanced mathematics, is due to vague or confused ideas about implication. Distressingly often, students have very definite, and entirely wrong views about it. One even finds completely incorrect statements on this matter displayed with emphasis in otherwise very fine textbooks. For these reasons it may be worth while to discuss common fallacies and ways of countering them.

One property of "H implies C" is accepted by all, namely the property that if H implies C and H is true, then C is true. In brief, *a conclusion that is implied by a true assumption is true.* This is the basic property of implication from which follows everything that can legitimately be said about it. The trouble arises from imagining things about implication that do not follow from this property and are indeed in conflict with it.

The relation that holds between H and C when H implies C is conveniently represented by an arrow $\rightarrow$. Then we read "$H\rightarrow C$" as "H implies C." This arrow symbol has the advantage of suggesting that the direction of reasoning is from the hypothesis H to the conclusion C. It helps distinguish sharply between $P\rightarrow Q$ and its converse $Q\rightarrow P$. With the aid of this symbol the fundamental property stated in italics in the previous paragraph may be represented in the following pattern of valid reasoning.

$$(1)\qquad \begin{array}{l} H\rightarrow C \text{ true} \\ H \text{ true} \\ \hline \therefore\ C \text{ true} \end{array} \qquad \textbf{Valid}$$

This pattern is used over and over again in elementary mathematics, both in geometric proofs and in deducing one equation from another in algebra.

A second valid pattern of reasoning is the following.

$$(2)\qquad \begin{array}{l} H\rightarrow C \text{ true} \\ C \text{ false} \\ \hline \therefore\ \text{H false} \end{array} \qquad \textbf{Valid}$$

This pattern is the basis of indirect proofs. Its validity can easily be seen from (1). For suppose $H\rightarrow C$ true and C false. If H were true, it would follow from (1) that C is true. Since C is false, this cannot be the case, and the only other possibility is that H is false. In brief, *an assumption that implies a false conclusion must be false.*

The above patterns seem reasonable to most people, but unfortunately many find the following invalid patterns acceptable.

$$(3)\qquad \begin{array}{l} H\rightarrow C \text{ true} \\ H \text{ false} \\ \hline \therefore\ C \text{ false} \end{array} \qquad \textbf{Invalid!!!}$$

$$\begin{array}{lll} & H \to C \text{ true} & \\ (4) & \quad C \text{ true} & \textbf{Invalid!!!} \\ \hline & \therefore H \text{ true} & \end{array}$$

According to (3), if an assumption is false, any conclusion derived from it is false. But *it is* not true *that a conclusion implied by a false assumption must be false.* According to (4), if a conclusion is true, any assumption that implies it must be true. But *it is* not true *that any assumption that implies a true conclusion must be true.* The fact is that a false assumption implies both true and false conclusions, and a true conclusion follows both from true and false assumptions. Perhaps the best way to convince the student of this is to give examples. Since it is easy to give examples where true conclusions follow from true assumptions, and false conclusions follow from false assumptions, we give here several examples in which true conclusions follow from false assumptions. (Of course, the fourth case of false conclusions from true assumptions is the one for which no examples can be given, and this is indeed the essential point in understanding the nature of implication.)

Example 1: H: $1{,}000{,}000=1$. *C:* $0=0$. Even the most "stubborn" student will probably admit that H is false and C is true. Now if $1{,}000{,}000=1$, then $1{,}000{,}000-1{,}000{,}000=1-1$, since equals can be subtracted from equals. But this is just $0=0$. In other words, we have proved that $H \to C$. It would be easy to make up any number of similar numerical examples in which a true conclusion can be derived from a false assumption. It would be instructive for students to experiment in order to see how *correct arithmetic can be used to derive either true or false conclusions from false assumptions, whereas from a true assumption only true conclusions can be derived.*

Example 2: H: In the triangle pictured in Figure 1, $\angle a=25°$, $\angle B=10°$, and $\angle C=145°$. *C:* $\angle A+\angle B+\angle C=180°$. From the figure H is evidently false. We know that C is true by a familiar theorem. Moreover, C follows from H by simply adding the angles! Again we have a case in which $H \to C$ is true, H is false, and C is true. Examples of this kind can be constructed *ad lib* by citing a case in which the hypotheses of a true theorem are satisfied, then taking C consistent with the conclusion of the theorem and H contrary to some aspect of the constructed case.

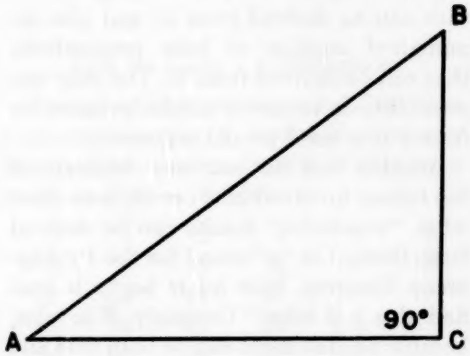

Figure 1

Example 3: H: The ocean consists entirely of grade-A milk. *C:* The ocean contains water. Our empirical knowledge makes it obvious that H is false and C true. But it is easy to show that $H \to C$. For suppose that the ocean is grade-A milk. It is well known that milk contains water. It immediately follows that the ocean contains water. It is easy to construct examples of this kind by making assumptions contrary to known facts and then reasoning from them until some true result is found.

Example 4: H: The New York Yankees won every game in 1953. *C:* The Yankees won the pennant in 1953. Again H is false, C is true, yet if the Yankees had won every game they certainly would have won the pennant.

Example 5: H: All people with red hair are very hot tempered. *C:* Johnny Jones is very hot tempered. H can be proved false by citing cases of red-headed people who are shy, retiring, and not at all excitable. Now let us suppose that Johnny Jones is

both red headed and hot tempered. Then C is true. Also H implies C, because if all people with red hair are very hot tempered, then Johnny Jones is very hot tempered.

The examples could be multiplied indefinitely. Given any true proposition, there is an unlimited number of false propositions from which it can be derived. Given any false proposition, there is an unlimited number of true propositions that can be derived from it, and also an unlimited number of false propositions that can be derived from it. The only impossibility is to derive a false proposition from a true one by valid argument.

Another way to convince doubters of the fallacy involved in (3) or (4) is to show what "wonderful" results can be derived from them. Let "p" stand for the Pythagorean Theorem. Now let H be "p is true and also p is false." Certainly H is false, because no statement can be both true and false. Now let C be "p is true." Certainly $H \rightarrow C$, because if two statements are both true then either one of them is true. Applying (3) on the false assumption that it is a valid pattern of reasoning, we conclude that C must be false, i.e., "p is true" is false, and hence p is false. So we have proved that the Pythagorean Theorem is false! More than that, the argument is equally good no matter what p is. Hence we have shown that if (3) is accepted as a valid pattern any statement can be proved false! Now if we let C' be "p is false," it is easy to see as above that $H \rightarrow C'$. Applying (3) again, we conclude that "p is false" must be false, i.e., p must be true. Hence by using (3) we could prove that any statement is true! In short, *the use of (3) would enable us to prove that any statement is both true and false!*

No wonder that these fallacies are the delight of those who wish to make a point regardless of its truth. All the demagogue has to do is to dress up his use of (3) or (4) so that his audience does not see the fallacy clearly, and he can carry the day. This is made easier by the fact that the fallacious patterns (3) and (4) are vaguely similar to the valid patterns (1) and (2). Indeed they can be distinguished only by understanding the difference between the two terms of an implication and the fact that implication is a relation directed from the hypothesis to the conclusion.

This writer has heard many discussions of whether mathematics has "transfer value." Here is a fundamental idea that plays an important role in all straight (and much crooked) thinking. There is no problem of transfer, since the application is direct and immediate. Examples can easily be found in every academic subject, in every trade, in the daily press, and in students' arguments. By driving home an understanding of the meaning of "if" and vaccinating students against invalid patterns of reasoning, the teacher can earn the gratitude of college teachers, employers, and the general public, all of whom have a vital interest in clear thinking. No single "mathematical fact" could be as useful to the student in later life as clarity on this basic tool of thought.

Words! Words! Words!

In Hamlet, Polonius takes a satirical jibe at Hamlet for reading mere words. However, Hamlet is not alone in being self-satisfied, seemingly, by use of words. Consider some arithmetic terminology of today.

Concrete number. Now number is an abstraction, it can be nothing else. So, we have a concrete abstraction. Can there be anything so foolish? It's like saying, "This white pussy cat is black." Actually what people mean when they use the term "concrete number" is that they have a concrete representation of a number. For example, * * * is a concrete representation of the number "3" but it is *not* a concrete number.

In the same way the term "abstract number" is a misuse of words. "Abstract number" is redundant. It says, in essence, "an abstract abstraction." Number is abstract. In this case it would be better to say, "A number without its representation."

Wouldn't the communication lines be much clearer if we clarified our terminology?

The factorgram[1]

KENNETH P. SWALLOW, *Newark College of Engineering, Newark, New Jersey.*

A simple proposition taken from the theory of numbers developed in such a way that the high school pupil cannot fail to be interested.

THE PROBLEM of finding all the prime numbers has intrigued mathematicians through the ages. The many attempts to solve this problem have yielded only methods that will produce a finite number of primes, the most noted of these being the Sieve of Eratosthenes. The Factorgram is an adaptation of this systematic mechanical method.

In the Eratosthenes Sieve, to find all the primes less than a selected number, N, all the integers from 2 to N are written in order. The number 2, which is known to be a prime, is encircled and every second number from 2 is crossed out. These are the multiples of 2 and hence cannot be primes.

2 3 ~~4~~ 5 ~~6~~ 7 ~~8~~ 9 ~~10~~ 11
~~12~~ 13 ~~14~~ 15 ~~16~~ 17 ~~18~~ 19 ~~20~~, etc.

The number 3, which is prime because it is the only remaining number less than 2^2, is encircled and every third number from 3 is crossed out.

2 3 ~~4~~ 5 ~~6~~ 7 ~~8~~ ~~9~~ ~~10~~ 11
~~12~~ 13 ~~14~~ ~~15~~ ~~16~~ 17 ~~18~~ 19 ~~20~~, etc.

Now, 5 and 7 are the only remaining numbers less than 3^2, therefore they must be prime numbers. This process is continued until every multiple of every prime number up to $\sqrt{N}$ is crossed out. The remaining numbers are the prime numbers less than N.

[1] Kenneth P. Swallow, "Elementary Number Theory in High School Mathematics," pp. 84–93, Unpublished Master's Thesis, Ohio State University, 1952.

HOW TO MAKE A FACTORGRAM

To find all the prime numbers less than a selected number, N, by the Factorgram, place all the numbers from 0 to N in rows of six numbers as follows:

0	1	2	3	4	5
6	7	8	9	10	11
12	13	14	15	16	17
18	19	20	21	22	23
24	25	26	27	28	29
30	31	32	33	34	35
36	37	38	39	40	41

Now the multiples of 2 can be crossed out by drawing lines through the entire first, third and fifth columns, with the exception of the number 2 itself. Similarly, the multiples of 3 can be crossed out by drawing a line through the entire fourth column, with the exception of the number 3 itself. The first column contains multiples of 3 but it is already crossed out. Next, the multiples of 5 are to be crossed out. The first six of these, 5, 10, 15, 20, 25, and 30, lie in a straight line running diagonally downward from right to left. The next six multiples of five (35 to 60) lie in another straight line, which is parallel to the first line. All the multiples of 5 can be crossed out by a set of such parallel lines. Next, the multiples of 7 can be crossed out by a set of such parallel lines running downward from left to right. The multiples of all prime numbers can be crossed out by similar sets of parallel lines. In the Factorgram as in the Eratosthenes Sieve, when all the multiples of all the prime numbers

less than $\sqrt{N}$ are crossed out the remaining numbers less than N are all primes.

The Factorgram can be made on a piece of paper and then rolled into a cylinder so that the numbers form a helical spiral. (In Figure 1, roll so that the two zeros coincide.) In this form, each of the sets of parallel lines which cross out the multiples of the prime numbers will also form a helical spiral.

FEATURES OF THE FACTORGRAM

The main purpose of the Factorgram, as of the Eratosthenes Sieve, is to find all the primes up to any selected number. However, the Factorgram has many features not found in the usual Sieve.

1. The mechanical process is very easy. The columns of numbers can be made quickly with a typewriter. If a long Factorgram is to be made, periods should be placed after the numbers as was done in Figure 1. The period, rather than the figure, is used to represent the exact location of each number. (In Figure 1 the distance from the zero line to each number is proportional to the magnitude of the number. This improves the Factorgram in its cylindrical form but is not really necessary for proper operation.) A pair of draftsman's triangles can be used to draw the parallel lines needed to cross out the multiples of each prime number. The first line of each set of parallel lines is determined by zero and the prime number. All such lines pass through zero, since zero is a multiple of every number.

2. The prime numbers, which seem to be so haphazardly scattered through the number system have, with the exception of 2 and 3, settled down to occupy positions in only two of the Factorgram's six columns.

3. The presence of prime pairs of the form p and $p+2$, such as 5 and 7, 11 and 13, etc., and of the form p and $p+4$, such as 7 and 11, 13 and 17, etc., become more obvious. Also, the relationships of prime numbers to the number 6 are emphasized.

4. Just as a prime number can be identified by the lack of lines passing through it, a composite number can be identified by the one or more lines passing through it. These lines provide a means, free of all trial-and-error methods, for finding all the prime factors of a composite number. It is this property of the Factorgram which gives it its name.

The method of factoring a composite number by the Factorgram is as follows: First locate the number on the Factorgram and trace any one of the lines passing through it back to its prime number origin (the last number on the line before reaching zero). This number is one of the prime factors of the original composite number. Divide the original number by this prime factor to obtain a second factor. Find this new factor on the Factorgram to see whether it is prime or composite. If it is prime, the problem is completed; if it is composite, continue the process until the factors of this factor are prime.

For a numerical example, consider the factoring of 117. There are two lines passing through it on the Factorgram. One of these goes back to 3. Dividing 117 by 3 we have 39. On the Factorgram, 39 has two lines passing through it also, one going to 3 and the other to 13. Therefore, the factors of 117 are $3\cdot 3\cdot 13$.

Frequently the divisions will be unnecessary because there may be as many lines passing through the given number as there are prime factors of the number. In this case, each line will give one of the prime factors. In fact, the necessity for division occurs only in two cases, (a) if the same number occurs as a factor two or more times, and (b) if one of the factors is greater than the square root of the largest number on the Factorgram. Variations of the Factorgram that will eliminate the division in both of these cases can be made, but these variations become overly complicated with too many lines.

5. If two numbers have a common factor, they will be connected on the Factorgram by the line representing that factor.

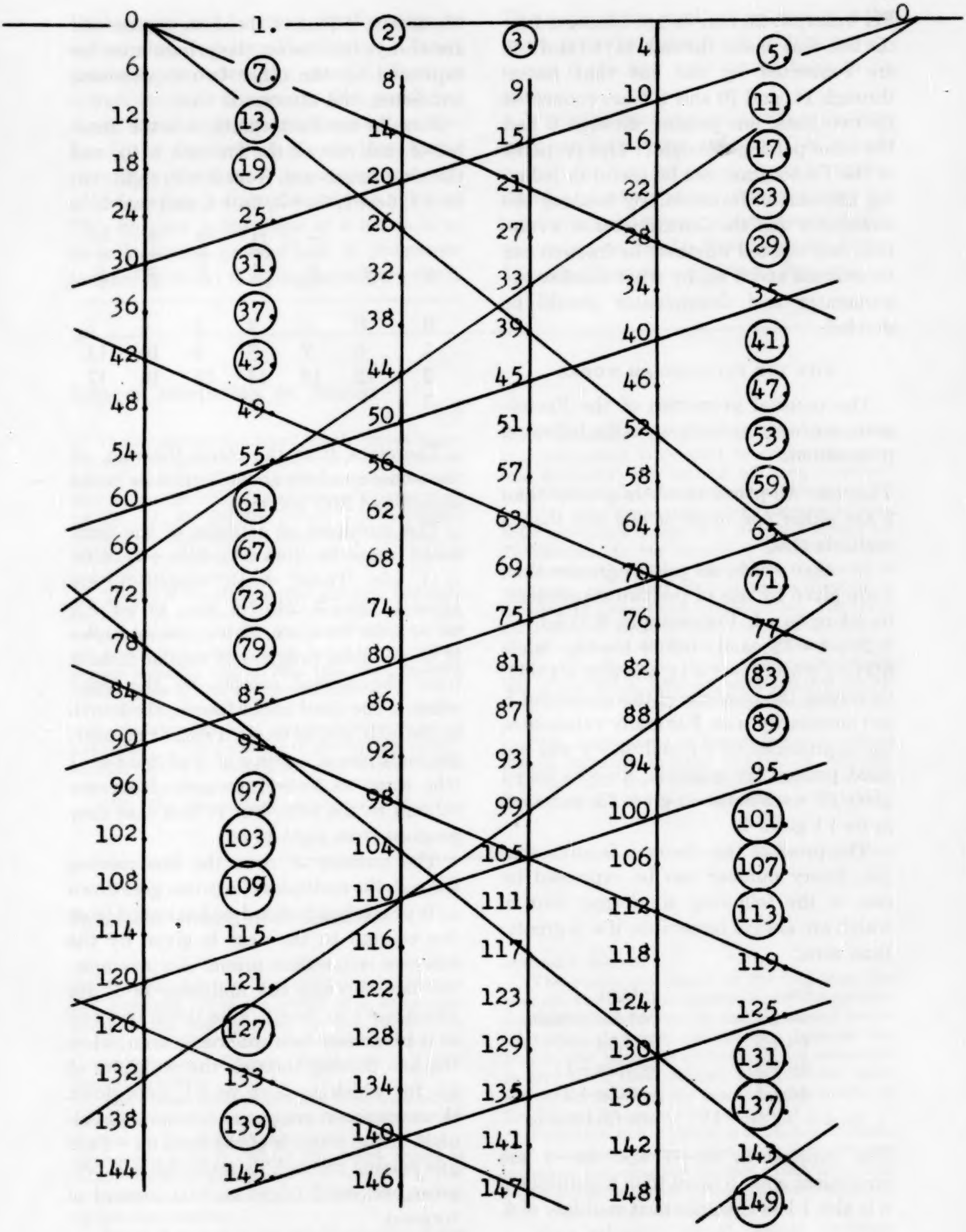

Figure 1. THE FACTORGRAM
(1 to 149)

For example, 26 and 65 are connected by the line that passes through 13; 88 and 121 are connected by the line that passes through 11; and 70 and 105 are connected by two lines, one passing through 5 and the other passing through 7. This property of the Factorgram can be useful in reducing uncommon fractions. By locating the numerator and the denominator of a fraction, one can tell whether the fraction can be reduced and if so, by what number the numerator and denominator should be divided.

WHY THE FACTORGRAM WORKS

The unusual properties of the Factorgram are based entirely upon the following proposition:

Theorem: All prime numbers greater than 3 are either one more or one less than a multiple of 6.[2]

In other words, all primes greater than 3 are given by one of the two expressions, $6n+1$ or $6n-1$. For example, $6(1)+1=7$, $6(2)+1=13$ and $6(3)+1=19$, while $6(1)-1=5$, $6(2)-1=11$, and $6(3)-1=17$. Of course, the converse of this statement is not necessarily true. For many values of n, the expressions $6n+1$ and $6n-1$ will not yield primes; for example, $n=4$ in $6n+1$ gives 25, $n=6$ in $6n-1$ gives 35, and $n=8$ in $6n+1$ gives 49.

The proof of this theorem is quite simple. Every number can be expressed by one of the following six forms, four of which are always factorable, if n is greater than zero.

$6n$	$=6(n)$
$6n+1$	not factorable
$6n+2$	$=2(3n+1)$
$6n+3$	$=3(2n+1)$
$6n+4$	$=2(3n+2)$
$6n+5$ (or $6n-1$)	not factorable

The expressions $6n+5$ and $6n-1$ are equivalent since 5 more than a multiple of 6 is also 1 less than the next multiple of 6.

[2] In congruence notation, for $p>3$, $p\equiv\pm1$ (mod 6).

Obviously, if four of these six expressions are always factorable, the primes must be expressed by the other two expressions, and hence, the theorem is true.

Now, in the Factorgram, n is the number of each row (if the first row is 0), and the six columns are, from left to right, $6n$, $6n+1$, $6n+2$, $6n+3$, $6n+4$, and $6n+5$.

n	$6n$	$6n+1$	$6n+2$	$6n+3$	$6n+4$	$6n+5$
0	0	1	2	3	4	5
1	6	7	8	9	10	11
2	12	13	14	15	16	17
3	.	.	.	.	.	.

Therefore, from the above theorem, all the prime numbers above 3 must lie in the second and fifth columns.

The multiples of a prime of the form $6n+1$ must be $2(6n+1)$, $3(6n+1)$, $4(6n+1)$, etc. These, when simplified, are $12n+2$, $18n+3$, $24n+4$, etc., or $6n'+2$, $6n''+3$, $6n'''+4$, etc. Hence, the multiples of $6n+1$ must progress in regular fashion from the second column to the third column, the third to the fourth, the fourth to the fifth, and so on. In a similar manner, the multiples of a prime of the form $6n-1$ (the same as $6n+5$) progress from one column to the next, but in this case they progress from right to left.

The number of rows the line passing through the multiples of a prime goes down as it progresses forward or backward from one column to the next is given by the value of n for that prime. For example, the line through the multiples of 7, for which $n=1$ in $6n+1$, goes down one row as it progresses forward one column, while the line passing through the multiples of 19, for which $n=3$ in $6n+1$, goes down three rows as it progresses forward one column. If the prime is of the form $6n-1$ the line passing through its multiples will progress backward (right to left) instead of forward.

This sort of slope is helpful both in setting up the parallel lines in making the

Factorgram and in using the Factorgram in factoring. In a long Factorgram it is not necessary to trace the parallel lines or spirals back to the prime that produced them. One merely needs to locate the number to be factored and note how many rows down the line (or lines) goes as it progresses forward or backward one column. This number is the value of n which is to be substituted in $6n+1$ if it progresses forward or $6n-1$ if it progresses backward. The value of the resulting expression is the same prime number that would be obtained if the line were traced back to its origin.

While the Factorgram is neither particularly profound nor useful, it is simple enough for high school students to understand and offers many opportunities for interesting classroom or mathematics club discussion as do the Eratosthenes' Sieve, Pascal's Triangle, and Magic Squares.

Science education in Russia

The newsletter published jointly by the Engineering Manpower Commission of Engineers Joint Council and Scientific Manpower Commission makes the following report on a paper read by Dr. M. H. Trytten before The American Society of Mechanical Engineers, meeting in Milwaukee, September 8–10, 1954. (See Newsletter #66, September 27, 1954.)

"There is the danger that growing familiarity with the details of Soviet progress in science and engineering will dull the senses of the American public to its significance, but it is hoped that serious meaning will be read into the following points made in Dr. Trytten's remarks:

"'1) The Soviet educational system begins with kindergarten, into which children enter at age three . . . the elements of reading are taught so that the child should be able to read upon entering the elementary school.

"'2) The elementary schools are of seven years' duration. . . . About 32% of the seven-year curriculum is devoted to arithmetic, algebra, geometry, the natural sciences, introductory physics, and the elements of chemistry.

"'3) After elementary school the student may enter secondary school or a technical school (or *Technikum*). In the secondary schools, largely preparatory for higher education, there are no electives, and about 40% of the curriculum is devoted to science and mathematics. The technical schools train technicians with subprofessional status in three- or four-year courses of study. Enrollment in these upper-grade schools was 4,600,000 in 1953–54, with 760,000 graduates, of whom 50,000 were subprofessional engineers.

"'4) Institutions of higher learning—about 900 in number—enrolled 916,000 in 1951–52. The fact that more than 50,000 engineers were graduated in 1954 has been widely publicized.'"

Facts for thoughtful teachers

A report on "America's Resources of Specialized Talent" has just been published by Harper and Brothers. It is the result of a three-year study which was financed by the Rockefeller Foundation. The study was conducted by the Commission on Human Resources and Advanced Training.

The whole report is destined to be a standard reference for those interested in America's resources so teachers should have a general acquaintance with it. While the entire report is worth teachers' attention, here are just a few of the salient conclusions and facts brought out by the commission.

Fewer than half of the top 25 per cent of the high school graduates in the United States earn college degrees; six out of ten of the top 5 per cent earn degrees.

The report comments on the fact that this country wants atomic power plants, atomic submarines, a cure for cancer, and bigger and better television. However, the country distrusts the people who have the ability and the education to give them these things. This distrust limits the extent to which the intellectual potential of the United States can be realized.

Guidance and a better articulation between high school and college courses are desirable as a means of reducing the hurdles which now tend to discourage capable youngsters from going on with their college education.

More extensive use of trained "womanpower" would go far toward relieving some of the shortages we now face.

Let's guess it first

SHRAGA YESHURUN, *Natanya, Israel, suggests a way to help pupils solve verbal problems. Teachers of algebra may wish to add this method to their repertoires.*

One of the most discussed, but as yet unanswered, questions in mathematical education is this:

How can we help pupils write equations adequate to solve verbal problems?

The age-old procedure has been to let x represent the number in question and then carry out on x all the operations stated in the problem. That this method causes pupils great difficulties is known by every experienced teacher.

Another widely used method is to translate the text of the problem into algebraic language. But this method, too, does not lessen pupil difficulties. After all, this way tends only to camouflage the abstract operations.

Why not try a concrete approach? Instead of operating on x as the problem requires, let the pupil guess a number as a possible candidate for the answer. On this supposed result the pupil then performs the operations implied by the working of the problem. Thus the pupil can check whether his guess is correct. If his guess is wrong, then the pupil replaces his guessed number by x, which provides him the equation he needs.

LINEAR EQUATIONS

To demonstrate this method with a simple example we shall take a problem leading to a linear equation with one unknown:

> The half, the third, and the quarter of a number make together 50. Find the number.

Transforming the verbal problem into an equation requires the following steps:

1. We guess the wanted number, making the guess as reasonable as possible. In the problem at hand, 12 obviously would be too small, whereas 120 or 1200 would be too large. On the other hand, it is desirable in this case to guess a number which is divisible by 2, 3, and 4, i.e., some multiple of 12. Such a choice facilitates the computations called for in the problem. For practical reasons (as we shall see later) it is desirable to guess a number which does not occur in any way among the data; in this case neither 2, nor 3, nor 4, nor 50 should be chosen.

Suppose that the guessed number is 60. It is entered in the first two columns of the following table:

Condition and its result	Original form	Check
The guessed number: 60	60	

2. In the second step we perform on the guessed number all the operations implied in the problem. The conditions, their results, and the original forms of the results are recorded.

If the problem is manifestly an application of a well-known theorem, then the data and the theorem serve in the next step to check the guess. If not, then a key number among the data enables the checking step. The key number, 50 in our example, is not listed in the table.

Now the table looks thus:

Condition and its result		Original form	Check
The guessed number:	60	60	
Its half:	30	$\frac{60}{2}$	
Its third:	20	$\frac{60}{3}$	
Its quarter:	15	$\frac{60}{4}$	

Note that only calculations on the guessed number are recorded under "Original form."

3. The most important step is the check of the guess. Here three possibilities exist:

(a) A theorem serves as a check.
(b) The key number, often the last number among the data, provides a check.
(c) Two procedures employing the guessed number produce the same result.

In the example at hand the last number in the data receives especial attention. Do $30+20+15=50$? Since $30+20+15$ does not equal 50,

$$\frac{60}{2}+\frac{60}{3}+\frac{60}{4}\neq 50.$$

Now the pupil replaces the guessed number by x and $\neq$ by $=$:

$$\frac{x}{2}+\frac{x}{3}+\frac{x}{4}=50$$

The reason for not using a number in the problem as the guessed number is clear enough: the pupil avoids a wrong replacement if he selects a number not given in the problem.

The result of the foregoing equation is $x=46.154$.

4. Now the hitherto empty column labeled "Check" provides a pattern for checking the answer to the problem. The final form of the table becomes:

Condition and its result		Original form	Check
The guessed number:	60	60	46.154
Its half:	30	$\frac{60}{2}$	23.077
Its third:	20	$\frac{60}{3}$	15.385
Its quarter:	15	$\frac{60}{4}$	11.539

Since $23.077+15.385+11.539=50.001$, the desired number is 46.154.

OTHER EQUATIONS

Problems leading to equations of the second degree can be translated into equations by similar procedures. Problems leading to systems of equations get almost the same treatment, except that:

1. Each unknown requires a separate guess.
2. There are as many checks involved as unknowns.
3. Several key numbers among the data, rather than one, are reserved for checking the guesses.
4. All results must check.

MIXTURE PROBLEMS

Here the amount and price of each kind of goods and the amount and price of the mixture are recorded in the table. When the pupil checks his guess he puts the values of the various goods together to compare with the value of the mixture; he also compares the total amounts of the items with the amounts of the mixture.

In problems dealing with alloys and solutions the pupil records in the table the weights of the constituents and their gross weight. The check of the guess determines whether the total weight of one constituent in the several original alloys matches the weight of that constituent in the composed alloy.

As an example:

How much alcohol of 80% and how much alcohol of 40% must be mixed to get 80 grams of alcohol of 70%?

Condition and its result	Original form	Check
The guessed number: 50g. of 80%	50	60
It contains 40g. of alcohol	$\frac{50\cdot 80}{100}$	48
Then there will be 30g. of 40%	80–50	20
It contains 12g. of alcohol	$\frac{(80-50)\cdot 40}{100}$	8
The composed alcohol is 80g. of 70%		80
It contains 56g. of alcohol		56

Check of the guess	Check of the result
$40+12\neq 56$	$60+20=80$
$\frac{50\cdot 80}{100}+\frac{(80-50)\ 40}{100}\neq 56$	$48+\ 8=56$
$\frac{80x}{100}+\frac{(80-x)\ 40}{100}=56$	

Whence $x=60$.

DIGIT PROBLEMS

In this type of problem the pupil records guesses for each digit involved and then lists the numbers thus resulting from the conditions of the problem.

As an example:

A two-digit number is 8 times as great as the sum of its digits. The number with the same digits in reversed order is 18 greater than the sum. Find the number.

Condition and its result		Original form	Check
Guess for the tens' digit:	3	$x=3$	7
Guess for the units' digit:	2	$y=2$	2
The original number:	32	$3\cdot 10+2$	72
The sum of the digits:	5	$3+2$	9
The tens' digit in the reversed number:	2	2	2
The units' digit in the reversed number:	3	3	7
The reversed number:	23	$2\cdot 10+3$	2
The sum of the digits:	5	$2+3$	9

Check of the guess	Check of the result
$32\neq 8\cdot 5$	$72=8\cdot 9$
$23=18+5$	$27=18+9$
$3\cdot 10+2\neq 8(3+2)$	
$2\cdot 10+3=18+(3+2)$	
$10x+y=8(x+y)$	
$10y+x=18+(x+y)$	

Whence $x=7$ and $y=2$.

ADVANTAGE OF THE GUESS METHOD

As the foregoing examples suggest, the method of guessing furnishes the pupil with definite steps to follow. Even those pupils rating no higher than average can successfully apply this method.

It has been the experience of the writer that pupils with average and poor previous progress tend to choose the guess method almost exclusively. Pupils with good previous progress tend also to employ the method, but they use it for the more complicated cases, when the formulation of equations causes some difficulty.

Many of our "teachers" are not really teachers. They are mathematicians, physicists, historians, linguists, etc.—not teachers. Many of them are men (and women) of great stature; major contributors to science, technology, and the arts; but they are not teachers. On some scales of worth to humanity they outweigh the teachers; but they are not teachers. They might even be indispensable to institutions of higher learning; but they are not teachers. To them, students are a means; to teachers, students are the end products—all else is a means. Hence there is but one interpretation of high standards in teaching: standards are highest where the maximum number of students—slow learners and fast alike—develop to their maximal capacity.—*Joseph Seidln*, "High Standards: Sacred and Profane," *Mathematics Magazine, March–April 1950, pp. 191–192.*

Research in mathematics education—1953

KENNETH E. BROWN, *Specialist, Secondary Schools Section, U. S. Office of Education, Washington, D. C.*

A report on research in mathematics education for 1953. This should answer some of your questions, for example, what are the best predictors of college success?

WHAT RESEARCH was carried on in mathematics education in 1953? What are the recent research findings directly related to the teaching of mathematics? To assist teachers in obtaining answers to these and similar questions, an attempt was made to summarize the research in mathematics education of the past calendar year.

In January 1954 a questionnaire was sent to 400 colleges that offered graduate work in mathematics education or whose staffs had made previous contributions in this field. From these inquiries were received approximately 300 answers including abstracts of 43 studies. The research included 27 masters' theses, 13 doctors' dissertations, and 3 studies by college faculty members. Seventeen of the studies were concerned primarily with mathematics education in the secondary schools, 14 with mathematics in the elementary schools, and 11 with college mathematical problems. The variation among the studies was more pronounced than the similarity. The topics varied from model ship-building in mathematics to the mathematics needed by coal-mine supervisors. Fifteen studies were on content and an equal number were primarily concerned with methods of instruction. However, these two categories are not mutually exclusive. Several studies involved both content and method. Seven studies were concerned with prognosis and evaluation.

A further examination of these research studies revealed that experimental research is still pronounced although both an experimental group and a control group may not be used. In 21 of the studies, pupils and classroom situations were involved. Further analysis of the research reveals an emphasis on the meanings of mathematical concepts and skills. Eleven of the studies contain discussions and specific suggestions for making the teaching of mathematics more meaningful. The authors of eight papers were trying to solve the problem of the poorly prepared pupil in mathematics. These papers centered around such topics as developing a college course in mathematics in which little or no mathematics is a prerequisite, developing a remedial program in high school mathematics, and a survey of what colleges are doing for the poorly prepared student in mathematics.

Some of the problems attacked by the persons engaged in mathematics education research in 1953 are indicated by the questions listed below. The number in the parentheses following the answers refers to the item in the bibliography upon which the answer is based.

If the project method is used, do the children make satisfactory progress in computation? In a recent study, it was concluded that quite a few children made little or no gain in computational arithmetic when the project method was used alone and that a supplementary program for mastery should be provided for at least the basic processes. (6)

Can the entire sixth grade course of study be covered by the project method? If several

projects of different types are carefully planned so that they involve all phases of arithmetic, it is possible. (6)

Is there a great need for more manipulative devices that can be used by teachers of arithmetic? A study of devices used by teachers in selected schools in Massachusetts indicated that the need was not for more devices but rather a better understanding by teachers of how to use existing devices most effectively. (30)

What effect do time of day and chronological age have upon arithmetic achievement at the fifth and sixth grades? There is none, according to a recent study. (40)

Do more girls than boys like arithmetic? A study of the attitudes of boys and girls in the fourth, fifth, and sixth grades indicated that there is little difference between the sexes in their attitudes toward the study of arithmetic. (2)

Are there differences in the extent to which first-grade children from different cultural and socio-economic groups understand money? A recent study indicates that there is no social class difference in money knowledge. In fact, a study points out that for many pupils the money lessons and exercises in the first grade only reiterate what they already know. (28)

How do elementary education majors compare with other college students in their proficiency in mathematics for general education? Elementary education majors had one of the lowest mean scores of all the groups sampled. (8)

How much college mathematics should be required of teachers of elementary school arithmetic? A recent doctoral study concludes that at least one year of college mathematics is necessary for preparation to teach in elementary schools. The study showed that teaching experience did not develop understandings although certain skills may be improved. The test results indicated that the greatest lack of knowledge is in the area of meanings. (11 and 34)

Is it possible to teach critical thinking to high school pupils? Critical thinking was taught to high school pupils and a test five months later indicated a large percentage of retention. (9)

When should the study of variation be introduced in geometry? It should be introduced early in the course. The study of variation is effective in leading students to discover some of the theorems in plane geometry. (12)

What place should the concepts of limit have in plane and solid geometry? Theorems or postulates involving the properties of limit should be omitted from plane and solid geometry on the secondary school level and all proofs involving limits should be informal. (18)

Does creative activity increase achievement in geometry? A group of pupils whose instruction was supplemented with creative activity not only made higher achievement scores but greater interest scores than the control group. (22)

What are the best predictors of college success? The most important predictors of success in all university studies are reading comprehension and achievement in high school mathematics. (4)

What is the best predictor for success in algebra? The best predictor for the final mark is the first six-weeks' algebra mark and the next best is the eighth grade arithmetic final mark. (24)

Are mathematics classrooms equipped to use multisensory aids? A study of 240 public schools in Oregon showed that in many cases they were lacking in adequate facilities conducive to the effective use of multisensory aids. Not only were the availability and use of spherical and graphic blackboards and blackboard equipment inadequate in many schools, but few supplementary mathematics books of a historical, recreational, or applicational nature were being used. (23)

What are the most troublesome problems for the beginning college mathematics instructor? The most frequent of the problems which are particularly troublesome to the beginning college mathematics instructor do not center on subject matter but on student-teacher relations (25).

What can be done about the poorly prepared college students in mathematics? A study based on a questionnaire from colleges in 46 states concluded that remedial classes are the best temporary solution. The best final solution is to cover less material in high school mathematics courses but cover it more thoroughly. (41)

A compilation of the summaries of research studies in mathematics education for 1953 was a part of the joint project of The National Council of Teachers of Mathematics and the United States Office of Education. The summaries include author, title of the study, year study was completed, sources from which a copy of the study may be obtained, the problems studied, procedures, findings, and conclusions. The compilation entitled *Mathematics Education Research Studies—1953*, Circular No. 377-II (free), may be secured from the Publications Section, Office of Education, Department of Health, Education, and Welfare, Washington 25, D. C. If research studies in mathematics education completed in 1953 have been omitted or if you have suggestions for improving these summaries, please notify the writer of this article or any member of the Research Committee.

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Erroneous arithmetical notions

"Shallow numerists," as Cocker is made to call them, have long been at work upon the question how to *mu'tiply* money by money. It is, I have observed, a very common way of amusing the tedium of a sea voyage; I have had more than one bet referred to me. Because an oblong of five inches by four inches contains 5×4 or 20 *square* inches, people say that five inches multiplied by four inches *is* twenty square inches: and, thinking that they have multiplied length by length, they stare when they are told that money cannot be multiplied by money. One of my betters made it an argument for the thing being impossible, that there is no *square* money; what could I do but suggest that postage-stamps should be made legal tender. Multiplication must be *repetition:* The repeating process must be indicated by *number* of times. I once had difficulty in persuading another of my betters that if you repeat five shillings as often as there are hairs in a horse's tail, you do not *multiply five shillings by a horsetail.*

I am very sorry to say that these wrong notions have found support—I think they do so no longer—in the University of Cambridge. In 1856 or 1857, an examiner was displaced by a vote of the Senate. The pretext was that he was too severe an examiner: but it was well known that great dissatisfaction had been expressed, far and wide through the Colleges, at an absurd question which he had given. He actually proposed such a fraction as

$$\frac{6s.3d}{17s.4d}.$$

As common sense gained a hearing very soon, there is no occasion to say more. In 1858, it was proposed at a college examination, to divide 22557 days, 20 hours, 20 minutes, 48 seconds, by 57 minutes, 12 seconds, and also to explain the fraction

$$\frac{321.18s.8d}{621.12s.9d}.$$

All paradoxy, in matters of demonstration, arises out of muddle about first principles. Who can say how much of it is to be laid at the door of the University of Cambridge, for not taking care of the elements of arithmetical thought?—*Taken from Augustus De Morgan: A Budget of Paradoxes. Vol. 11, p. 251. Italics are the editor's.*

Magic letters—TV— and magic squares

ROBERT V. ESMOND, *Director, Radio–TV, Northern Illinois State Teachers College, De Kalb, Illinois.*

It takes lots of time and effort to put on a thirteen-minute TV program. The experience of one group at Northern Illinois State Teachers College will prove to be of interest to others who are thinking about a mathematical TV program

THE MAGIC LETTERS—TV—have touched Northern Illinois State Teachers College, De Kalb, transforming it from a school with vague television hopes for the unfocused future to a school which now produces television programs. Figuring in this transformation was Northern's mathematics department, a department which accepted and successfully met the challenge of television.

This television venture by Northern's mathematics staff—and by the college in general—warrants special attention, for it represents the kind of TV situation now facing, and soon to face, the myriad of educational and cultural groups which cannot operate television stations of their own.

In brief, this was the situation. Last summer Northern was approached by officials of WREX-TV, Channel 13, in Rockford, Illinois, and invited to participate in the programming of that station as a public service feature. The college accepted and assigned the development of television programs to the Regional Services Division. Under my direction, volunteer representatives from college departments and administrative offices met in committee session to review TV ideas and select from them those which would best meet the needs of Northern's first series of television programs.

In view of the fact that NI's programs were to be 15 minutes in length and presented every second Tuesday evening, the college soon appreciated the fact that a formal course presentation or anything like it could not be attempted. A good public relations impression, through the presentation of representative academic activities from the college, became the purpose of the series.

With more college departments and groups expressing interest in presenting a program than program periods available, a five-member advisory group, selected from the large faculty committee on television, reviewed program ideas and recommended them for development into programs.

Northern's Mathematics Department, represented by Herbert Miller of the department staff, suggested "Magic Squares," a program dealing with number phenomena which had the necessary popular appeal and still was truly a sampling of one mathematics activity in the preparation of teachers. The TV advisory group recommended the idea for development, and active preparation got under way.

Miller's greatest problem, he soon found, was determining what to include and what to omit in the presentation. The Mathematics Department had narrowed its TV program possibilities down to magic

squares. Yet the subject contained facets which could be pursued at length. In addition to a number of books on recreational mathematics, the *Encyclopedia Americana* gives the subject of magic squares four pages, and *Encyclopaedia Britannica* treats it on three.

Thirteen minutes on magic squares seemed an incredibly short time for a presentation, and that's really all the time there was after allowing time for the opening and closing college format announcements. Yet that was the challenge of this and all other television programs in the college series: to know just how much to include, how to say it, how to show it, and when to stop.

Miller, like all members of the faculty a newcomer to this medium, did the research and prepared the content of the telecast. From his classes be selected three mathematics majors, two seniors and a junior, to assist him in the presentation. Under his guidance the program developed.

Their first step was a "read-through" of the material on magic squares, with Miller and his students checking the timing. Some trimming was necessary. Careful advance planning had, however, eliminated the necessity of major cuttings.

Two weeks before the program was to be presented, I met with the group, watched them rehearse—without props—and answered the students' excited questions: "What shall we wear?" "Do we need special makeup?" and so forth. Their timing was good, very good for a program still ten days away from broadcast. It remained to be seen whether the use of the necessary props would appreciably alter the timing.

One week to go, and now the intensive work began. The Fine Arts Department contributed sheets of gray-toned cardboard on which the magic squares could be drawn in black. Gray-toned or cardboard with some tone to it is preferred to white for television work, eliminating the possibility of an objectionable black-on-white contrast. Miller and his students decided that construction of the magic squares before the camera eye would take up too much valuable program **time.**

Program materials were kept to a minimum and included an artist's easel, a Dürer print containing a magic square, a pointer, a felt inking pen, and a half dozen pieces of cardboard on which the magic squares were prepared.

Program time was 6:45 p.m., Class A viewing time, so far as the telecaster is concerned. WREX, interested in developing itself as a community service station, took early steps to bring civic and educational groups from the area into its programming. The station has the enlightened policy of granting a portion of its best viewing time—Class A—to Northern and other similar groups. In return it can demand that its public service programs be high in quality. Northern appreciates the confidence placed in it by the station, shown by placing the college's "Four Good Years" series between a network newscast and a network sports program, and has demanded quality presentation from its program groups.

The "Magic Squares" program group left the campus in mid-afternoon and ar-

Mathematics majors from Northern Illinois State Teachers College explain the Lo-shu magic square during a recent television program.

rived in Rockford—45 miles away—in time to "talk through" the presentation with the production director. Cameramen were briefed on angles, and the production director instructed the floormen on the setting for the program.

Before their turn came, Miller and his students had time to watch several "live" programs. Week after week, that watching experience has proved of value to Northern's program groups, for it has given them opportunity to project themselves into the spirit of TV presentation, to dispel any "butterflies" (this may not happen until they're on camera), and to become aware of the work of the floormen and camera men moving about taking their shots, throwing cues, and giving time signals.

Northern's "Four Good Years" series has a standard opening and closing used with each program. The program opens with the series' title superimposed over motion pictures of campus scenes. In the background is *a cappella* singing by the college choir, a tape recording prepared by the Music Department well before the series got under way. As the music fades, the credit card is supered, I read a standard introductory announcement, my camera comes "hot" and the program is under way.

" . . . and here to open our program is Mr. Herbert Miller of the Mathematics staff," I said, concluding my introduction.

As the camera dollied back from Miller, he commented on magic squares as a form of recreational mathematics and said that this program represented an aspect of the training with which the student preparing to teach mathematics might come in contact.

Seated on a couch across from Miller were his three student helpers, Victoria Brefka, Don Lund, and Bernadette Gucwa. The camera spotted each student with a close up as each was introduced.

The teacher-student relationship was there, but in an atmosphere of comfortable informality, with the enthusiasm of each person for his part in the program carrying through.

Miller felt some historical background was necessary in discussing magic squares, and each student added his bit in tracing this evolution. Miss Brefka opened the demonstrations, explaining the Lo-shu, believed to be the oldest known example of a magic square. It dates back to the Emperor Yu in China in the twentieth century B.C., and, according to legend, it was found on the back of a turtle. As the camera moved in on the Lo-shu for close-up work, the second camera was ready to cut to Lund on the couch for his explanation of how, through algebra, one can determine the magic constant of a square of any size when the cells are occupied by consecutive numbers.

Miss Brefka had pointed out that the Lo-shu represented the only possible nine-celled square using the first nine numbers. Lund stepped to the easel and followed through with a card showing the algebraic equation for determining the magic constant of any given magic square and a table of the constants for squares from three to twelve cells in a side.

As the camera cut to Miss Gucwa on the couch, she told how the magic square entered our western culture. She held a print of Albrecht Dürer's engraving, "Melancholia," and invited the viewers to "look more closely" at the magic square in one corner of the print. The camera eye accommodated and brought that tiny portion of a $5'' \times 7''$ engraving into living rooms across 25 counties of northern Illinois and Wisconsin.

While Miss Gucwa concluded her remarks with a reference to the De la Loubère method of constructing magic squares, Miller, off camera, readied the cards for the next demonstration. Miss Brefka picked up the discussion and, in addition to explaining De la Loubère's method, demonstrated how one is constructed according to this method. Similarly, Lund told about De la Hire's method.

As each card was presented, one camera moved in for close-up work, the second remaining back to cut quickly to the group.

To keep the end of the program flexible, permitting cutting or stretching as time indicated, Miller prepared a number of variations of magic squares, such as those made with dominoes, bordered squares, and a Nasik square.

As Miller concluded the program, he offered viewers a bibliography of materials used in preparing this TV program. In the next day's mail was a letter indicating the general response to and acceptance of "Magic Squares." It read, in part:

"Will you please send me five copies of your paper on 'Magic Squares'? I would like to pass these out to four teachers in our PTA group and have one copy for myself.

"We have placed your programs on our recommended list. Our parent group enjoys them very much and we wish we could have such worth-while programs more often."

"Such worth-while programs" are not come by easily, as Miller and his mathematics students can attest. From hours of research and preparation of program materials, from six hours of active rehearsal, from the acceptance and carrying out of many individual responsibilities came "Magic Squares." There was nothing magic about the success the program has enjoyed in campus and community circles. Hard work assured that success.

The "Magic Squares" experience of the Mathematics Department at Northern will not remain unique for long. As the so-called local or community television stations develop, other mathematics groups across the country will find television opportunities and challenges readily available to them. Though the mathematics area may not possess all the possibilities for showmanship inherent in the physical sciences, and so necessary to TV work, mathematics has a share in this new medium. "Magic Squares" showed that. The program possibilities fire the imagination. The television opportunities are on the way. Together, they offer a vital challenge to mathematics teachers everywhere.

"There never has been in the world's history a period when it was more worthwhile to be a teacher than in the twentieth century; for there was never an age when such vast multitudes were eager for an education, or when the necessity of a liberal education was so generally recognized."—*William Lyon Phelps*

"If I had a child who wanted to be a teacher, I would bid him Godspeed as if he were going to a war. For indeed the war against prejudice, greed and ignorance is eternal, and those who dedicate themselves to it give their lives no less because they may live to see some fraction of the battle won. They are the commandoes of the peace, if peace is to be more than a short armistice. As in a relay race, our armed men have handed victory to those who dare not stand still to admire it, but must run with it for very life to a further and larger goal."—*James Hilton*

Edited by Phillip S. Jones, University of Michigan, Ann Arbor, Michigan

"A new ballad of Sir Patrick Spens"

Professor Norman Anning first called the editor's attention to "A New Ballad of Sir Patrick Spens" appearing in *Q Anthology*, a selection from the prose and verse of Sir Arthur Quiller-Couch.[1]

This poem is a parody upon an old Scottish ballad. Its appearance here is justified by the fact that it largely deals with the first proposition of Book I of Euclid's *Elements*. This first proposition was *To describe an equilateral triangle upon a given finite straight line*. Our Figure 1 is the diagram for this proposition. The construction for the proposition is given in the poem, and the proof we leave to the reader who may, if he chooses, look it up in Todhunter's *Euclid's Elements*. This famous English textbook, which is referred to in the third stanza of the ballad, is available today as No. 891 of Everyman's Library sold in this country by E. P. Dutton and Company.

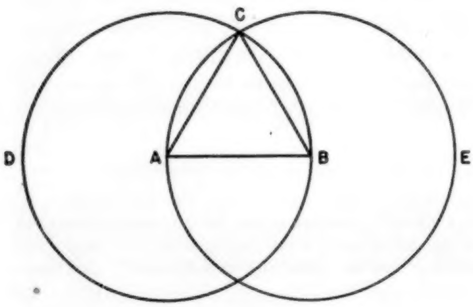

Figure 1

Incidentally, the significance of this proposition is that it serves as the basis for Proposition 2 which shows how any given length may be laid off from a given point in a given direction without transferring it from the first location to the second by means of compasses. Euclid's compasses were "collapsible," i.e., he did not assume that lengths could be translated.[2]

The "Asses' Brigg" cited in the fourth from the last stanza refers to the fifth proposition of Book I of Euclid's *Elements* which states that the base angles of an isosceles triangle are equal. This proposition was the English schoolboy's "pons asinorum" either because Euclid's diagram looked like a bridge or because the asses in the geometry class had trouble crossing the bridge to more advanced work.

The original ballad of which this is a parody told the story of Sir Patrick Spens (or Spence, or Skipper Patrick) who was ordered by King James of Scotland to sail to Norway. Sir Patrick complained to himself about whoever recommended him to the king as the nation's best sailor, but he obeyed orders unquestioningly in spite of foul weather and forebodings and sailed

[1] *Q Anthology*, compiled and edited by F. Brittain (New York: The Macmillan Co., 1949).

[2] A diagram for this second proposition, further discussion of Euclid's restriction on the "picking-up" of lengths and its relationship to proofs by superposition will be found in a note in The Mathematics Teacher for March, 1952 (vol. XLV, p. 232 ff.). Todhunter's *Euclid's Elements* cited above will also shed light on this question.

(to bring home the king of Norway's daughter?).

On the return voyage the ship foundered in a storm. Sir Patrick Spens and all the fine Scotch lads with him perished.

The ballad may have a historical origin dating back to when Margaret, daughter of Alexander III, was married in 1281 to Eric, King of Norway. The ballad appears in early manuscripts and other sources with many variations. Some music for it has been preserved.[3]

A NEW BALLAD OF SIR PATRICK SPENS

The King sits in Dunfermline toun
Drinking the blude-red wine:
'O wha will rear me an equilateral triangle
Upon a given straight line?'

O up and spake an eldern knight,
Sat at the King's right knee—
'Of a' the clerks by Granta side
Sir Patrick bears the gree.

'Tis he was taught by the Tod-huntére
Tho' not at the tod-hunting;[4]
Yet gif that he be given a line,
He'll do as a brave thing.'

Our King has written a braid letter
To Camgrigge or thereby,
And there is found Sir Patrick Spens
Evaluating π.

He hadna warked his quotient
A point but barely three,
There stepped to him a little foot-page
And louted on his knee.

The first word that Sir Patrick read,
'Plus x' was a' he said:
The neist word that Sir Patrick read,
'Twas 'plus expenses paid.'

The last word that Sir Patrick read,
The tear blinded his e'e:
'The pound I most admire is not
In Scottish currencie.'

Stately stepped he east the wa',
And stately stepped he north:
He fetched a compass frae his ha'
And stood beside the Forth.

Then gurly grew the waves o' Forth,
And gurlier by-and-by—
'O never yet was sic a storm,
Yet it isna sic as I!'

Syne he has crost the Firth o' Forth
Until Dunfermline toun;
And tho' he came with a kittle wame
Fu' low he louted doun.

'A line, a line, a gude straight line,
O King, purvey me quick!
And see it be of thilka kind
That's neither braid nor thick.'

'Nor thick nor braid?' King Jamie said,
'I'll eat my gude hat-band
If arra line as ye define
Be found in our Scotland.'

'Tho' there be nane in a' thy rule,
It sall be ruled by me';
And lichtly with his little pencil
He's ruled the line AB.

Stately stepped he east the wa',
And stately stepped he west;
'Ye touch the button,' Sir Patrick said,
'And I sall do the rest.'

And he has set his compass foot
Untill the centre A,
From A to B he's stretched it oot—
'Ye Scottish carles, give way!'

Syne he has moved his compass foot
Untill the centre B,
From B to A he's stretched it oot
And drawn it viz-a-vee.

The tane circle was BCD,
And ACE the tither:
'I rede ye well,' Sir Patrick said,
'They interseck ilk ither.'

'See here, and whaur they interseck—
To wit, with yon point C—
Ye'll just obsairve that I conneck
The twa points A and B.

[3] *The English and Scottish Popular Ballads*, edited by Francis James Child (Boston: Houghton, Mifflin and Company, 1885). Vol. II, pp. 17–32.
[4] "Tod-hunting" meant fox-hunting.

'And there ye have a little triangle
As bonny as e'er was seen;
The whilk is not isosceles,
Nor yet it is scalene.'

'The proof! the proof!' King Jamie cried:
'The how and eke the why!'
Sir Patrick laughed within his beard—
''Tis ex hypothesi—

'When I ligg'd in my mither's wame,
I learn'd it frae my mither,
That things was equal to the same,
Was equal ane to t'ither.

'Sith in the circle first I drew
The lines BA, BC,
Be radii true, I wit to you
The baith maun equal be.

'Likewise and in the second circle,
Whilk I drew widdershins,
It is nae skaith the radii baith,
AB, AC, be twins.

'And sith of three a pair agree
That ilk suld equal ane,
By certes they maun equal be
Ilk unto ilk by-lane.'

'Now by my faith!' King Jamie saith,
'What plane geometrie!
If only Potts had written in Scots,
How loocid Potts wad be!'[5]

'Now wow's my life!' said Jamie the King,
And the Scots lords said the same,
For but it was that envious knicht,
Sir Hughie o' the Graeme.

[5] Potts was another Cambridge mathematician contemporary with Todhunter.

'Flim-flam, flim-flam!' and 'Ho indeed!'
Quod Hughie o' the Graeme;
''Tis I could better upon my heid
This prabblin prablem-game.'

Sir Patrick Spens was nothing laith
When as he heard 'flim-flam,'
But syne he's ta'en a silken claith
And wiped his diagram.

'Gif my small feat may better'd be,
Sir Hew, by thy big head,
What I hae done with an A B C
Do thou with X Y Z.'

Then sairly sairly swore Sir Hew,
And loudly laucht the King;
But Sir Patrick tuk the pipes and blew,
And played that eldritch thing!

He's play'd it reel, he's played it jig,
And the baith alternative;
And he's danced Sir Hew to the Asses' Brigg,
That's Proposetion Five.

And there they've met, and there they've fet,
Forenenst the Asses' Brigg,
And waefu', waefu', was the fate
That gar'd them there to ligg.

For there St. Patrick's slain Sir Hew,
And Sir Hew Sir Patrick Spens—
Now wasna' that a fine to-do
For Euclid's Elemen's?

But let us sing Long live the King!
And his foes the Deil attend 'em:
For he has gotten his little triangle
Quod erat faciendum!

This apparent impermanence of theory has produced an impression in many able minds that science is fundamentally unstable, forever casting off the old in favour of the new; but this is to misunderstand the scientific method. One may as well reject a child because it is growing up.—*O. G. Sutton, Mathematics in Action (London: G. Bell and Sons, 1954).*

Edited by Lucien B. Kinney, Department of Education, Stanford University, Stanford, California

The aspirations of this department

by Lucien B. Kinney

Editor's Note: This is a new department of THE MATHEMATICS TEACHER. Dr. Kinney has kindly consented to be the editor of this department. Your ideas will be welcomed.

The mathematics program in the junior high school has the same major purposes as at any other level. These have been well analyzed by the Post-War Commission in two general categories:

"To insure mathematical competence for the ordinary affairs of life, to the extent that this can be done for all citizens as a part of general education"; and

"To provide sound mathematical training for future leaders in science, mathematics, and other learned fields."

Through the grades and high school these constitute the guiding purposes for mathematics teaching, and the basis on which results must be evaluated.

Yet it is also true that in designing the mathematics program, at any given age level, the characteristics of the pupils must be taken into account. To a considerable extent they determine how the purposes of mathematics are to be achieved. To some extent they may even indicate concomitant purposes for the program. Probably at no other level are these pupil characteristics more significant than in the junior high school.

Much has been written about the characteristics of pupils in grades seven, eight, and nine. The literature on adolescence affords useful insights on the theoretical level. But on the practical side, the problems and opportunities created by the unique characteristics of pupils in the junior high school, whether these grades are organized as a separate unit or as part of an 8-4 plan, still present a challenge to the most expert teacher of mathematics.

We need more information on practices that have been tested and proved in the classroom; a clearinghouse for exchanging ideas and experiences. It is the purpose of this department to provide these services.

Many teachers (and former teachers, including the writer) consider that teaching mathematics in the junior high school is more exciting, and at the same time more exacting, than at any other level. The program of instruction in mathematics for these grades, as revealed by textbooks, syllabi, and reported classroom practices, tends increasingly to take into account the characteristics of the pupils for whom it is designed. It will be useful, accordingly, in initiating this department, to give some consideration to three functions that have become especially important in program and practices at the junior high school level in taking these characteristics into account.

1. *Attention to individual differences.* In the junior high school these differences reach their peak, although they have been increasing in amount throughout the grades. Pupils, for example, with I.Q.'s of 100 and 117 who differ in mental age by only one year at age 6, differ in mental age by two years at age 12, as they enter the junior high school. In the senior high school the differentiated curriculums will reduce somewhat the range of differences

in any one class. In the junior high school the major outcomes are still those of general education, rather than specialized preparation, and the classes are less homogeneous. Hence, in these years, the greatest ranges in such characteristics as these occur: interests and plans, abilities, probable future needs, personality, experiences, sex, home background.

Such variations within each class make practically every pupil a special case. In providing for these differences it is necessary to:

1. Make the work real, interesting, and important to pupils of a variety of backgrounds and plans.
2. Provide for the slower pupils by:
   (a) Careful regulation of rate of progress
   (b) Continued help with fundamentals and problem solving
   (c) Keeping problem situations concrete, leading from familiar to new
   (d) Careful attention to vocabulary
   (e) Variety in the fields of mathematics used—arithmetic, algebra, geometry
3. Provide for rapid pupils by:
   (a) Problem situations requiring greater insight, ingenuity, ability to organize an attack
   (b) Progressing further into mathematical abstractions
   (c) Challenges to solution of more difficult problems

2. *Orientation and guidance.* Perhaps the function most uniquely typical of junior high school mathematics is that of orientation and guidance. It is also one of the most important, since, on leaving the junior high school, each pupil will need to make his choice among the various curriculums of the senior high school. Each, in most school systems, should be prepared to make an intelligent decision between algebra and general mathematics in the ninth grade. If this is to be an intelligent choice, considerably more should be known about the ability and plans of the pupil than is commonly the case at present.

Not only the pupil, but society as well, has an important stake in the intelligence of this choice. This is becoming clear as the shortage of trained leaders in our society becomes more acute each year. The shortage in two of these leadership roles—teachers and scientific leaders—has forced itself on the attention of everyone. The same shortages exist in all leadership roles, however, and they emphasize the general responsibility of the junior high school teacher to discover and conserve talents and aptitudes needed in our society.

The specific responsibility of the mathematics teacher, as indicated by the Post-War Commission, is the shortage of engineers and technical scientists, amounting to about 155,000 at the most recent estimate, with no relief in sight. The engineering senior of 1954 was in the ninth grade in 1946. Only the fact that he had become interested in engineering, had recognized his interests and aptitudes in mathematics and science, and was guided into the required program at the crucial time, made his engineering career possible. How many potential engineers were lost at that point we cannot say, but the prevailing laissez-faire attitude toward the pupil's decision justifies the belief that the loss is considerable. In view of the dependence of our world leadership on our technical manpower, the function of orientation and guidance in the junior high school takes on a new significance.

We, in this country, are committed to the idea that each individual is free to choose his own career, in the light of adequate information about the nature, requirements, and rewards of the vocation, and of his own aptitudes, abilities, and interests. For the mathematics teacher, this means an exploration of the fields of mathematics and their vocational applications in grades 7 and 8. Recent texts and syllabi show how this can be done in such a way that the pupil can sample the fields of

algebra, geometry, and trigonometry, with the result that teacher and pupil both have some evidence to serve as the basis for the decision that usually must be made at the beginning of grade nine.

This exploration of vocations, the fields of mathematics, and of the aptitudes and interests of the pupil, is in line with the growing concern of pupils of this age with the nature and problems of adult society. Along with this is an increasing awareness in the individual pupil of the adult role he is about to play, socially and vocationally, and concern as to his own qualifications for these social and vocational roles.

Like all problems arising out of pupil characteristics, this is much more complex in practice than in theory. The wide range of individual differences mentioned above is especially evident here. Pupils differ widely, both in nature and amount, as to knowledge about adult society, vocations, and their own aptitudes. The study of social applications of mathematics offers opportunity not only as a response to such interests, but also as a means for creating them.

3. *The transitional function.* Each pupil in the junior high school is confronted with the major task of developing the self-direction and competence for independent effort that will be required for success in high school and college. Each year, through the grades, he has had the continuous attention of one teacher who, in most instances, was responsible for not more than thirty to forty other pupils. In the high school he will have to adjust to several teachers, each of whom has contact with 100 to 200 different pupils in his classes. In college, the relationship is still more impersonal, and the responsibility for success or failure rests with the pupil himself.

It is true that this is only one aspect of the adolescent task of acquiring independent adult status. Yet this does not justify a "sink or swim" policy. It emphasizes rather the share each junior high school teacher has in helping to develop effective study habits and ability to budget time, and to define personal standards of workmanship for each pupil. This major responsibility of the junior high school has been termed the *transitional* function.

In mathematics, the pupil needs to become self-reliant, adopting a systematic approach in attacking problems, and showing ability to identify and diagnose his difficulties. With appropriate experiences under careful guidance, this quality can be developed. There is good reason to believe, on the basis of research evidence, that meaningful learning promotes self-direction. By and large, it is a matter of carrying the ideas and procedures of meaningful learning into the junior high school, and extending them to fields other than arithmetic.

The general characteristics of the program. It is to be emphasized that we have been discussing the special functions of junior high school mathematics. In addition, of course, are the over-all purposes of mathematics teaching for general education and special preparation that were defined by the Post-War Commission. It is the influence of all these functions that has given junior high school mathematics its outstanding characteristics.

The program in mathematics in the junior high school that has developed in response to these influences has been well described by the National Committee on Mathematical Requirements as "an introductory, basic, exploratory course, in which the simple and significant principles of arithmetic, algebra, and geometry, statistics, and numerical trigonometry are taught so as to emphasize their natural and numerous relations." While varying in nature from one school to another, curriculum and instruction in junior high school mathematics today represent an effort to recognize the pupil needs peculiar to the junior high school population, and the purposes of mathematics in the curriculum.

Aspirations of this department. This brief review of functions and characteristics of junior high school mathematics

was undertaken as a survey of ideas and problems that this department should be concerned with. To be effective, the audience participation procedures of this department will require your help, as a junior high school teacher, in two ways.

First, we need to know what your special problems are on which practical suggestions are needed. If the contents of this department are to be useful, they should be concrete, specific, and to the point. The question is, what point? The same difficulties that are bothering you are bothering many others. It is to these that we should direct our attention.

In the second place, we need reports on your effective practices, activities, and materials of instruction. Visits to the classrooms of many teachers have convinced this writer that many expert practices are going on that have never been made available to the profession. The general discussion above was designed to outline some of the major areas where ideas are needed, but no good ideas on junior high school teaching are foreign to our interests. What have you tried that was effective? What were you trying to do? How did you go about it? What worked, and how well?

Let us hear from you.

What's new?

BOOKS

SECONDARY

Applied General Mathematics, Edwin B. Piper, Randolph S. Gardner, Preston E. Curry, New Rochelle, New York, South-Western Publishing Company, 1954. Cloth, viii+566 pp.

The Slide Rule, J. N. Arnold, New York, Prentice-Hall, Inc., 1954. Cloth, viii+206 pp., $3.40.

Trigonometry, Elbridge P. Vance, Cambridge 42, Massachusetts, Addison-Wesley Publishing Company, Inc., 1954. Cloth, viii+158 pp., $3.00.

COLLEGE

2222 Review Questions for Surveyors, Russell C. Brinker, Box 153, Blacksburg, Virginia, R. C. Brinker, 1954. Paper, 169 pp., $3.00, single copy; $2.50, 5 or more.

MISCELLANEOUS

Science Awakening, B. L. Van Der Waerden, English Translations by Arnold Dresden, with additions of the author, Groningen, Holland, P. Noordhoff Ltd., 1954. Cloth, 306 pp., $5.00.

The Microphysical World, William Wilson, New York, Philosophical Library, 1954. Cloth, vii+216 pp., $3.75.

The Psychology of Invention in the Mathematical Field, Jacques Hadamard, New York, Dover Publications, Inc., 1954. Cloth, xiii+145 pp., $2.50; paper $1.25.

DEVICES, FILMS, AND EQUIPMENT

Decimal Scales (#172)
Yoder Instruments, East Palestine, Ohio.
Wooden ruler with brass edge; 12" long; graduated in tenths of a foot with hundredths and half-hundredths as subdivisions, also graduated in centimeters and in inches with tenth of an inch subdivisions; lot of ten rulers, $1.30.

Income Tax Teaching Kit
Public Information Division, Internal Revenue Service, Washington 25, D. C.
Kit of materials for instruction in completing Form 1040, Short and Long, and Form 1040F; includes teacher's handbook; free.

Rubber Fraction Pies (#N230)
Creative Playthings, Inc., 5 University Place, New York 3, New York.
Set of eight 7" diameter rubber pies ($\frac{3}{8}$" thick); set of eight pies, $6.50.

Scientific Makit
W. R. Benjamin Co., Granite City, Illinois.
Construction set of 48 wooden balls, 16 wooden wheels, 192 wooden rods, and accessory parts; packed in metal box with hinged tray; postage prepaid; $5.00 or $5.50 west of Denver.

Vernier Calipers (#325)
Yoder Instruments, East Palestine, Ohio.
Caliper rule with sliding indicator; inside and outside capacity 5"; $1.50.

Weights and Measures
Encyclopaedia Britannica Films, Inc., 1150 Wilmette Avenue, Wilmette, Illinois. 16 mm film; black and white; 14 min.; collaborator, John R. Clark.

• MEMORABILIA MATHEMATICA

Edited by William L. Schaaf, Brooklyn College, Brooklyn, New York

With this, the first issue of Volume 48, a new department modestly makes its debut: "Memorabilia Mathematica"—noteworthy or memorable items of mathematical interest.[1] We shall wander in and out of the byways of mathematics: recreational items; humor; anecdotes; expository mathematics; the antiquarian's notes; gleanings from recent books; current events in the mathematical world; mathematical personalities and biographical notes; mathematical education past and present. We make no apologies for the selection of these contributions, endeavoring only to add something worth while and just a bit different.

What is a mathematician? . . .

If one were to ask: what is a botanist? what is a geophysicist? or, what does a paleontologist do? the answers are plain enough. But what is a mathematician? what does *he* do? Are mathematicians, as Stephen Leacock once said, born that way? Let us see what some of them have to say about themselves. To begin with, according to Sawyer:[2]

> Many people regard mathematicians as a race apart, possessed of almost supernatural powers. While this is very flattering for successful mathematicians, it is very bad for those, who, for one reason or another, are attempting to learn the subject.

Deservedly or not, this aura of mystery concerning mathematicians appears to be well-nigh universal and apparently ineradicable. Thus, from the pithy pen of the late S. Brodetsky:[3]

> Specialized skill always impresses the layman. . . . But if there is a specialty that arouses the deepest admiration, a skill that evokes the greatest wonder, this specialty and this skill are those of the mathematician.

> The mathematician is supposed to be a person endowed by nature with a special faculty. He plays with numbers as children play with wooden bricks, erecting complicated numerical and geometrical monuments whose structures only fellow-mathematicians can follow. He juggles with symbolistic abracadabras, and tosses about the notorious x, y, z with diabolic skill. He pastures in meadows of logarithmic tables, delves into spaces that convey no impression of reality, invents processes that smell of the witch's caldron, and enjoys a particular delight in getting zero for his final answer.
>
> What is a mathematician? How does his brain work? He is evidently a clever person—no non-mathematician would dare to suggest that a mathematician is not clever—yet he is a somewhat peculiar person, whose views on the practical affairs of life can be treated with a smile of mingled indulgence and amusement. He enjoys the adding up of long columns of figures, yet he is proverbially incapable of adding up two and five correctly. He can say the most wonderful things about space, yet he should be put in charge of a policeman when he wishes to cross the street.

In a somewhat more serious mood, we note the reflections of two outstanding scholars, one a historian, the other a mathematician:

> One soon realizes that mathematicians are much like other men, except in the single respect of their special genius, and that that genius itself has many shapes and aspects. . . . The great mathematician may be a man of very limited experience and wisdom outside his own field and his advice in non-mathematical matters may be of very little value; he may be burdened with all

[1] The new department supersedes the former "References for Mathematics Teachers." All three of the articles in this issue were written by Mr. Schaaf.

[2] W. W. Sawyer, *Mathematician's Delight* (Baltimore: Penguin Books, 1946), p. 1.

[3] S. Brodetsky, *The Meaning of Mathematics* (London: Ernest Benn, Ltd., 1929), pp. 5–6.

kinds of passions and weaknesses; in short, he is like the rest of us except in one essential respect.[4]

It is usual to exaggerate rather grossly the differences between the mental processes of mathematicians and other people, but it is undeniable that a gift for mathematics is one of the most specialized talents, and that mathematicians as a class are not particularly distinguished for general ability or versatility.[5]

Another characterization, at once whimsical yet cogent, comes in the form of an anecdote related by one of America's most distinguished contemporary mathematicians.[6] The story runs somewhat as follows: the great mathematician

. . . Liouville inspired William Thomson, Lord Kelvin, the famous Scotch physicist, to one of the most satisfying definitions of a mathematician that has ever been given. "Do you know what a mathematician is?" Kelvin once asked a class. He stepped to the board and wrote

$$\int_{-\infty}^{+\infty} e^{-x^2}dx = \sqrt{\pi}.$$

Putting his finger on what he had written, he turned to the class. "A mathematician is one to whom *that* is as obvious as that twice two makes four is to you. Liouville was a mathematician."

In recent years the role of the pure mathematician in modern industry has assumed more than passing significance. The distinction between typical mathematicians, on the one hand, and typical physicists and engineers, on the other, is interestingly suggested by an eminent authority[7] in the relatively new field of industrial mathematics, as follows:

[4] George Sarton, *The Study of the History of Mathematics* (New York: Harvard University Press, 1936), pp. 22–23.

[5] G. Hardy, *A Mathematician's Apology* (New York: Cambridge University Press, 1940), pp. 9–10.

[6] E. T. Bell, *Men of Mathematics* (New York: Simon and Schuster, 1937), p. 452.

[7] Thornton C. Fry, "Industrial Mathematics," (Bell Telephone Laboratories, June 1941), pp. 2–3.

The typical mathematician feels great confidence in a conclusion reached by careful reasoning. He is not convinced to the same degree by experimental evidence. . . . Because of this confidence in thought processes the mathematician turns naturally to paper and pencil in many situations in which the engineer or physicist would resort to the laboratory. For the same reason the mathematician in his "pure" form delights in building logical structures, such as topology or abstract algebra, which have no apparent connection with the world of physical reality and which would not interest the typical engineer; while conversely the engineer or physicist in his "pure" form takes great interest in such useful information as a table of hardness data which may, as far as he is aware, be totally unrelated to any theory, and which the typical mathematician would find quite boring.

A second characteristic of the typical mathematician is his highly critical attitude toward the details of a demonstration. For almost any other class of men an argument may be good enough, even though some minor question remains open. For the mathematician an argument is either perfect in every detail, in form as well as in substance, or else it is wrong. There are no intermediate classes. He calls this "rigorous thinking," and says it is necessary if his conclusions are to be of permanent value. The typical engineer calls it "hair splitting," and says that if he indulged in it he would never get anything done.

The mathematician also tends to idealize any situation with which he is confronted. His gases are "ideal," his conductors "perfect," his surfaces "smooth." He admires this process and calls it "getting down to essentials"; the engineer or physicist is likely to dub it somewhat contemptuously "ignoring the facts."

A fourth and closely related characteristic is the desire for generality. Confronted with the problem of solving the simple equation $x^3-1=0$, he solves $x^n-1=0$ instead. Or asked about the torsional vibration of a galvanometer suspension, he studies a fiber loaded with any number of mirrors at arbitrary points along its length. He calls this "conserving his energy"; he is solving a whole class of problems at once instead of dealing with them piecemeal. The engineer calls it "wasting his time"; of what use is a galvanometer with more than one mirror?

Women in mathematics . . .

Mathematicians, as teachers, have traditionally provided the training in mathematical techniques needed in scientific, technological and other professional occupations. The principal role of the modern mathematician is still that of the teacher, although in recent years mathematicians have been increasingly sought as research workers and consultants in an amazingly wide range of scientific, technical and industrial problems.

Of no little interest is the position of

women in mathematics in the United States today, as set forth in *Manpower Resources in Mathematics*, a study conducted jointly by the National Science Foundation and the United States Department of Labor Bureau of Labor Statistics.[8] The report concerns the status of some 2400 mathematicians, of whom about 1450 held the Ph.D., 750 held the master's degree, and about 200 others. Women represented only 8% of the mathematicians with Ph.D.'s, but nearly twice as large a fraction (15%) of those without doctorates. The per cent of women was highest (17%) among the mathematicians who had achieved but not gone beyond the master's degree

Women who hold professional positions in the field of mathematics are, with rare exceptions, employed by educational institutions. An extremely high proportion (94%) of the women Ph.D.'s studied were on the staffs of colleges and universities, and another 4% were in other educational institutions. Of the women with less advanced training, the proportion employed by colleges and universities was smaller (71%), but 15% were working for other educational institutions, chiefly high schools. Only a few had jobs in industry, government, or other types of employment.

The concentration of personnel in teaching, indicated by these figures, was greater in the case of women than with men. Among Ph.D.'s, for example, the proportion of teachers was 93% for women and 79% for men. Only 5% of the women Ph.D.'s held positions in the growing field of mathematical research, compared with 16% of the men.

[8] National Science Foundation, *Manpower Resources in Mathematics* (Superintendent of Documents, U. S. Government Printing Office, Washington 25, D C., 1954). Price 20 cents.

"Provision for individual differences"

Several times during the past few years I have had occasion to read the following excerpt to prospective secondary mathematics teachers, without first divulging its source:

> Mathematical studies have long held in the school curriculum the assured place which is due to one of the most distinctive activities of the human mind, of fundamental importance in both the intellectual and the practical field. Pure mathematics is, however, highly abstract and formal in character, and it is natural that these characteristics have given rise to difficulties in the school treatment of the subject. The traditional treatment has been not ill-suited to the ablest pupils, who have shown their capacity to profit from the discipline it can afford, but for the less well-endowed the academic rigours must be tempered. Recognition of this necessity has already found practical expression in many schools which have adapted courses to the varying capacities of their pupils. This process of adaptation, however, has not yet been applied widely enough, and the need for a greater differentiation of courses has become still more pronounced with the general acceptance of the principle that promotion to the secondary stage should in the main be determined by age rather than by attainment.

The ideas expressed in this quotation invariably seemed so familiar to my listeners —so much a part of the local scene—that their astonishment was considerable upon learning that this is the opening paragraph of a brief report on *Mathematics in Secondary Schools*, issued by the Scottish Education Department, published in Edinburgh in 1950.

My readers may be interested to pursue this document a bit further. The second paragraph reads:

> Differentiation has taken the form not only of adjusting the various syllabuses to suit the capacity of the pupils, but also of modifying their content in varying degrees to relate them, as appropriate, to the needs of life. Such practical references are not only important in themselves but they can also be a powerful factor in enlisting the interest of those who do not re-

spond readily to a more academic approach. Thus, in the syllabus intended for the weakest pupils, mathematics covers little more than the simple arithmetic of everyday life; in the syllabuses for the average pupils a treatment is encouraged which would relate mathematics to the life of the adult world in its technical, commercial, domestic, and civic aspects; and in the syllabus for the ablest pupils it is sought to combine this broader approach with the more rigorous treatment required as a foundation for advanced study.

So it would seem that learners differ from one another in much the same way everywhere, and that the teacher of mathematics is beset with the problem of providing for individual differences equally on either side of the Atlantic. One calls to mind the universal nature of mathematics; it appears that mathematical education, also, has about it an aura of universality.

Have you read?

MacCurdy, Robert D., and others. "Can We Teach Them How to Study?" *The School Review,* 62: 357–360, September 1954.

Throughout the years students have been told to study their lessons but only rarely have we given them any help in learning how to study. Watertown High School, Massachusetts, became concerned with their situation and proceeded to study the problem. This article is the story of their experiment. They selected two comparable groups and one was given instruction on how to study while the other was not. They developed both texts and tests. Were the results significant? They will surprise and I think inspire you to experiment in your school, not only on how to study but on many other phases of instruction and learning. There are many implications in this article.

Iyer, Venkachalan R. "The Hindu Abacus," *Scripta Mathematica,* 20: 58–63, March to June, 1954.

You and your students are all familiar with some form of the abacus; therefore, I am sure you will want to read this short article on the Hindu abacus written by a man who uses one. Did you know this system of calculation is still in use by all good Hindu Pandits? They never record their thoughts on paper. Did you know that this use of the abacus always produces the final result for all preceding steps? But why not get yourself a board, about 100 "cowies" or sticks to represent the digits, and about a dozen buttons to represent the zeros and do some calculations? You will want your students to read this article for the explanation of the processes of addition, multiplication, subtraction and division, and then they'll enjoy working some problems on their own.

Van Voorhis, W. R. "An Integrated Approach to Basic Mathematics," *The Journal of Engineering Education,* 44: 600–605, June 1954.

Here is another engineer who maintains that mathematics is the sustaining structure of engineering. But if it is to function as it should, much more integration than is commonly practiced must be carried out. General mathematics is sometimes poorly interpreted but in 1941, of 25 schools, 5 had integrated mathematics and 20 had a conventional program. In 1954, 16 of the 25 had an integrated program while only 9 still used the conventional one. Do you agree that planned integration will provide more motivation; that it will assist in available applications; that it will more easily fit into related courses; and that it will encourage and develop rigorous and analytic thinking? Do you believe that a higher level of efficiency will be reached; that the student will develop more self-confidence; and that more efficient methods will be selected? Before you answer these questions you should read this thought-provoking article.

Dutton, William H. "Measuring Attitudes Toward Arithmetic," *The Elementary School Journal,* 55: 24–31, September 1954.

Every day you have probably wondered just what your students think of mathematics and if your teaching has in any way affected their attitude toward mathematics. Mr. Dutton's study has attempted to find an answer to this question. You will want to read the details of his study, which started by the gathering from advanced students in the University of California over a period of five years, statements about their likes and dislikes in arithmetic. These statements were sorted, analyzed and scaled, then administered to 289 students of graduate, senior, junior and sophomore levels. It is encouraging to note that 67% favored arithmetic while only 11% strongly opposed it. It is liked because it challenges, satisfies, and is definite; but disliked because it is not understood, not properly timed, and not properly taught.

The author believes he has a device to measure attitudes, that these attitudes are developed in the classes, and that certain aspects of the subject have different effects. But you should read this yourself and make your own decision. —Philip Peak, *Indiana University, Bloomington, Indiana.*

A column of unofficial comment

The teacher shortage

by John R. Mayor, University of Wisconsin, Madison, Wisconsin

The nation is faced with the unhappy prospect of a very serious teacher shortage, one which is already felt in many places. The shortage promises to be most acute in mathematics and science because of the demand for those trained in these areas in industry and government. As just one indication of the seriousness of the situation, the American Association for the Advancement of Science is giving special assistance and attention to efforts of the A.A.A.S. Cooperative Committee on the Teaching of Science and Mathematics in developing and carrying out an Action Program intended to assist in alleviation of the shortage.

The Cooperative Committee has been asked to plan three sectional meetings using the theme "The Crisis in Science Education" for the annual convention of the American Association for the Advancement of Science, to be held in Berkeley, California, during the Christmas holidays. The proposed Action Program will be the principal topic for the three sectional meetings and the Action Program will also be presented to the Board of Directors for approval. An important phase of the program is concerned with encouraging university and college departments of science and mathematics to recognize a greater responsibility for programs of teacher education and in-service activities of schools.

The problems created by the teacher shortage should be of concern to every mathematics teacher. Almost certainly, classes in mathematics will be larger and those inadequately trained to teach will be given mathematics classes. In many schools teachers may be given additional teaching hours. It is likely to become more difficult for teachers to give the time necessary for satisfactory class preparation, for use of the best teaching aids, for attention to individual differences and interests, and desirable leadership in the school and community.

There is little value, however, in a cry of alarm unless it is accompanied by constructive suggestions. In preparation for this crisis mathematics teachers should

a. Support The National Council of Teachers of Mathematics and their state and local mathematics teachers groups.

b. Encourage students qualified by personality and ability for teaching to enter the teaching profession.

c. Live as a person who brings great credit to the profession and who, by his example, encourages others to enter the profession.

d. Support agencies working for recruitment of teachers.

e. Help the public understand the necessity of maintaining or improving present standards.

Edited by Emil J. Berger, Monroe High School, St. Paul, Minnesota

Simple paper models of the conic sections

Contributed by Ethel Saupe, Tracy, Minnesota

In second year algebra courses students are normally introduced to the conic sections by being told that certain curves which may be represented by quadratic equations in two unknowns are, in fact, equations of conic sections. In solid geometry the same students are told that a section of a right circular cone made by a plane is a conic section. Various kinds of models are occasionally employed to illustrate what the conic sections look like in the geometric sense, but ordinarily little effort is made to tie up the fact that a relation exists between the introduction given in algebra and the models presented in solid geometry.

The examples offered in the following paragraphs suffer somewhat from oversimplification and are certainly specialized, but they can be used as a convenient means for relating the graphs of quadratic equations in two unknowns and conic sections. In addition, every reader should be able to complete the models because the developments suggested are planned so that only paper is needed.

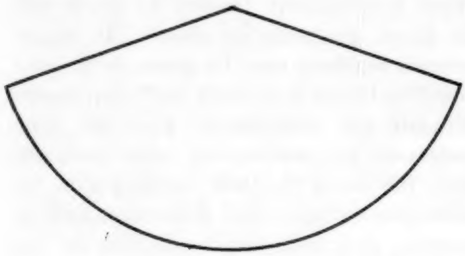

Figure 1a

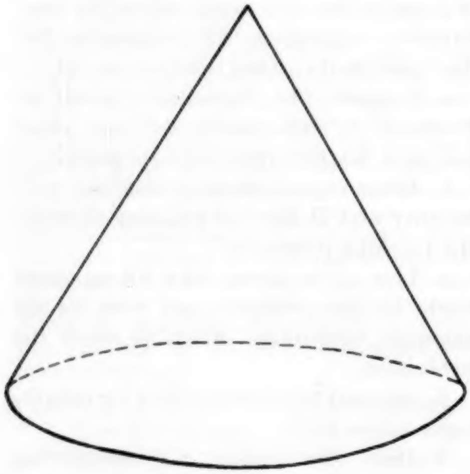

Figure 1b

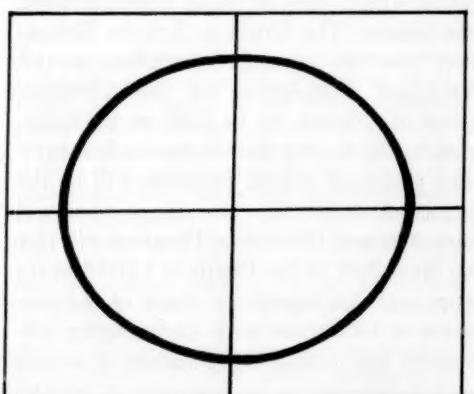

Figure 1c

It is probably not necessary to demonstrate that the circle is a conic section, but students sometimes doubt that the ellipse as developed in a graph has this property. Figures 1a–1d suggest the steps that may be followed to provide a concrete illustration of the concept in question. A right circular cone, made from a sector of a circle, will serve the purpose for which it is needed nicely (Fig. 1a, 1b). Draw a graph of an equation such as

$$x^2+4y^2=144,$$

or

$$x^2+25y^2=400,$$

cut out the area enclosed by the curve, and the materials are ready to be used for a demonstration of the fact that the curve is a conic section (Fig. 1c, 1d).

To illustrate that the parabola is a section of a cone made by a plane parallel to an element, lay the cone on one side, mount the graph on a stand with adjustable legs and fit it on the cone (Fig. 2a–2c). If the graph is mounted on a card before the area on the concave side is cut away the plane of the curve will be sturdier than if only the graph paper is used. A simple frame for the parabola can be made by punching holes in the corners of the card with an ordinary paper punch and inserting small rolled up pieces of paper to

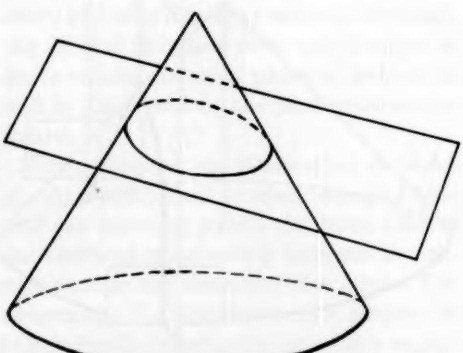

Figure 1d

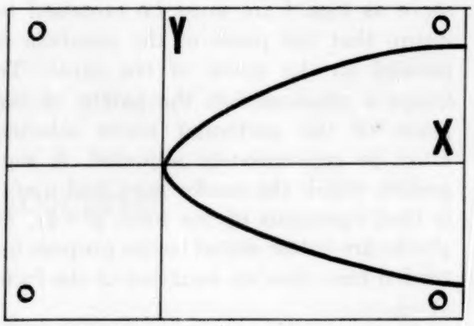

Figure 2a

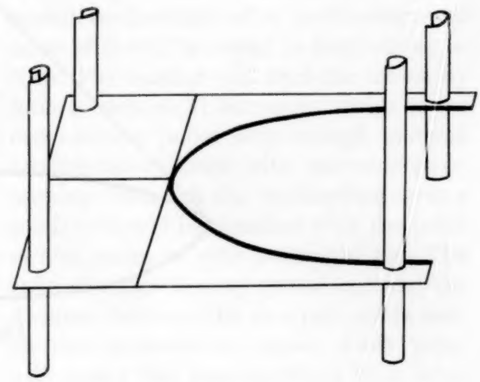

Figure 2b

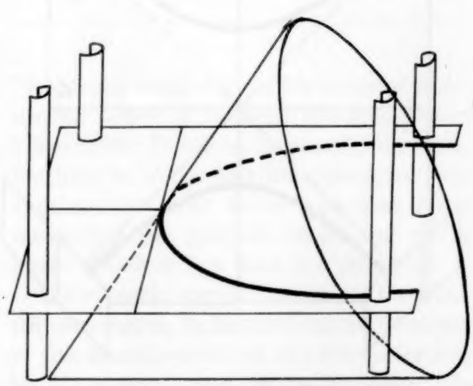

Figure 2c

serve as legs. Care must be exercised to insure that the plane of the parabola is parallel to the plane of the table. To locate a good section the height of the plane of the particular curve selected must be appropriately adjusted. A suggestion which the reader may find useful is that equations of the form $y^2=4x$, or $y^2=3x$ are better suited to the purpose intended here than an equation of the form $y^2=x$.

To construct a simple paper model for demonstrating that the hyperbola is a conic section, graph a curve of the form $y^2-x^2=a^2$ and form the cone with a sector having an angle of 255° (Fig. 3a–3c). If this suggestion is followed the asymptotes of the curve will be perpendicular and a section through the axis of the cone (when $a=0$) should be two perpendicular lines. The axis of the cone and the plane of the curve must, of course, be parallel to effect any hyperbolic section. Since the easiest way of arranging this situation is to have both cone and plane in vertical position, it is necessary to supply the plane (which carries the cut away curve) with feet (Fig. 3b).

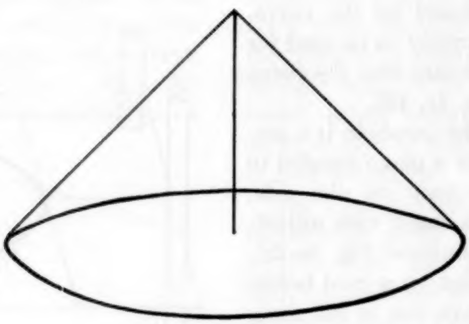

Figure 3a

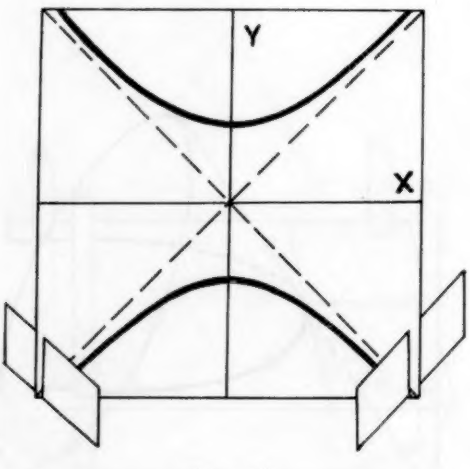

Figure 3b

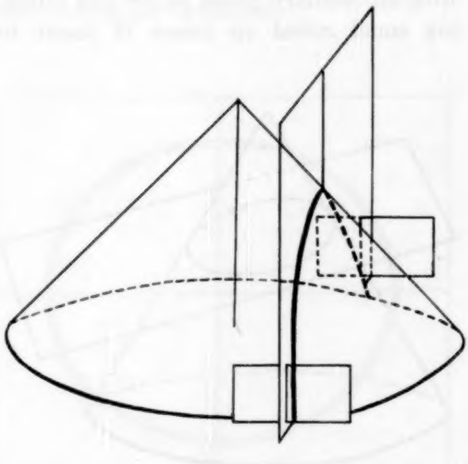

Figure 3c

• TIPS FOR BEGINNERS

Edited by Francis G. Lankford, Jr., University of Virginia, Charlottesville, Virginia

Helping pupils to make discoveries in mathematics

By Francis G. Lankford, Jr.

Young people learn mathematics best when they are taught to discover for themselves ideas basic to the subject. They learn least effectively when these ideas are told to them by the teacher as bits of abstract information to be learned. Incidentally, pupils are quick to identify a teacher who uses most of the classroom time in telling the pupils about mathematics. They are quite likely to say of such a teacher, "She would be a good teacher if she would only not talk so much. We would learn more if she gave us more time to think things out for ourselves." The method of teaching mathematics by discovery is sometimes called the laboratory method because it is similar to the inductive method used in the laboratory by the scientist. To be employed most successfully, this method frequently requires the use of some simple equipment. However, an inductive approach to the teaching of mathematics does not always require equipment of a physical nature, as will be indicated in the illustrations that follow.

First, consider an illustration in which physical equipment is used. Imagine that you are teaching pupils the basic idea of the constant relationship between the circumference and diameter of a circle. The assignment for tomorrow may simply be to ask pupils to bring the necessary equipment to class. This equipment will include a small cylinder for each pupil—a tin can, a small piece of cylindrical wood, even a piece of blackboard chalk. Each pupil will also need strips of cross-section paper. Preferably some of this paper will be scaled in fractions of a centimeter, and some of it will be scaled in fractions of an inch. The teacher will start the lesson by having each pupil tear or cut strips of the cross-section paper long enough to wrap around his cylinder with the ends overlapping. Through the overlapping ends a small hole will be punched with the point of compasses or with a straight pin. The strip of paper is now opened out and the distance between the two pin points read on the cross-section paper. Each pupil will record his measurements in a table arranged as follows:

	Inches	*Centimeters*
C		
D		
C/D		

Next, each pupil will use his strips of cross-section paper to measure the diameter of his cylinder. It will be necessary, of course, for him to make this measurement very carefully in order to be sure that he is measuring the greatest chord and not a chord which is less than the diameter.

When each pupil has made the four measurements, he should find the quotient of his measurement of the circumference by his measurement of the diameter. His first discovery will likely be that the two quotients are very nearly equal. Next, the

teacher will ask each pupil to announce the quotient he has found. The teacher will write these quotients on the blackboard. In all, there will be twice as many quotients as there are pupils in the class.

The second likely discovery is made when pupils observe that all of these quotients come very close to the value, three. Indeed, if any quotient was found very different from three, the teacher is justified in discarding this value and asking the pupil to measure again. Now the pupils are ready to write in their own language the discovery which has been made. It is highly important that each pupil be given freedom individually to express the idea which he has discovered. These expressions at first are likely to be awkward ones and need considerable refinement. When pupils have written and refined their own statements of the discovery they have made, the teacher may direct them to the page in the textbook where the rule or generalization is printed in boldface type. Pupils often experience considerable thrill at finding that the statements they have written on their own agree with those placed in the book by the author.

Now it should be clear in this illustration that it was not the purpose of the lesson to teach pupils that $\pi = 3.1416$ as a piece of factual information useful in solving word problems. Rather, the purpose of this lesson was to help pupils understand the constant relationship between the circumference and diameter of a circle. It is of small concern that the findings made by the pupils through their measurements did not produce an exact value of π. If we are thinking about a teacher in the junior high school grades, we will suggest that she simply tell the pupils the exact value of the relationship which they have discovered. If we are thinking about a teacher with a class in plane demonstrative geometry, we suggest that she follow the kind of laboratory lesson outlined, with a more abstract explanation of the constant relationship between the circumference and diameter of a circle.

A second illustration will deal with helping pupils understand the nature of direct variation and how to express it. A teacher may start by asking a question such as this, "If gasoline is priced at 30 cents per gallon, how much does it cost for one gallon? for two gallons? for five gallons? for ten gallons? for twenty gallons? Upon what does the cost of gasoline depend when the price is 30 cents per gallon?" This relationship may be expressed by the formula $C = 30n$ where C equals cost in cents and n equals number of gallons purchased. The teacher may now follow with such a question as this, "If an airplane travels at the rate of 300 miles per hour, how far will the plane go in one hour? in two hours? in three hours? in ten hours? How may we write as a formula the relationship between the distance a plane will travel at 300 miles per hour and the number of hours it travels?" This question may be followed with one which asks, "In a 20 per cent sale, how much discount will a person receive on an article priced before the sale at $5? on an article priced before the sale at $10? on an article priced before the sale at $40? How may you express in a formula the relationship between the discount in a 20 per cent sale and the price of an article prior to the sale?" Other illustrations may be used but soon the teacher is ready to help pupils make the discovery that there are many examples of two variables which change so that an increase in one will produce a uniform increase in the other or a decrease in one will produce a uniform decrease in the other. This simple idea is now given the name *direct variation*. Moreover, pupils are helped to see that in every formula which has been written on the board, expressing the direct relationships considered in the several examples, the same form is used. This form expresses one variable as equal to a constant times the second variable. This discovery may be expressed in words as well as in symbols.

It may be observed that this lesson, like the earlier one dealing with the circle, re-

quired much response from the pupils. The teacher asked questions in order. This is in direct contrast to a teaching procedure in which the teacher tells pupils that there is such a thing as direct variation, and that direct variation may be expressed in a rule which he states or in a formula which he writes. In the developmental lesson described above, pupils do much more than listen to the teacher's explanation. They participate actively in the evolution of an idea. Usually after an idea has been developed, application is in order. The many good exercises found in textbooks can be used for the necessary practice in applying the idea which has been acquired in a developmental fashion.

There are several implications of a teaching procedure directed toward helping the pupil discover mathematical ideas. First, discovery implies that the goal is not known by the learner at the outset of the journey. Moreover, his journey may start with some problem to be solved for which no ready solution is available to him. This does not mean, of course, that all problems will call for the discovery of an idea as yet unfamiliar to the learner. Frequently his problems will be solved by using concepts entirely familiar because they have been previously discovered. Moreover, a teacher may make a discovery problem of such an example as that of finding the ratio of the circumference to the diameter of a circle simply by suggesting that a relationship exists and directing pupils to try to find it. A second implication of a procedure that helps pupils discover mathematical ideas is that the pupils must be given freedom to make assumptions or to suggest ways of performing an operation. This simply means that teachers must accept the fact that the thinking of pupils engaged in making a mathematical discovery will doubtless not follow the orthodox pattern which has come to be accepted by the adult or mature student. They must be prepared for quite unexpected assumptions and suggestions. In fact, it may be that a teacher should feel that the extent to which she gets unorthodox but reasonable assumptions and suggestions from pupils as they engage in the discovery of an idea or operation, her pupils are really engaging in independent thinking. It is the teacher's responsibility, of course, to avoid mere trial-and-error learning. A third implication of a procedure emphasizing pupil discovery is that when a group of pupils have settled on what they regard as the true idea or relationship, or have decided on the best operation to employ, they must be given freedom in describing their findings. This is only to remind us that there are more ways than one for expressing a rule or of wording a definition. Incidentally, it seems that such freedom for pupils to write their own rules and definitions may help them a lot in building a working vocabulary in mathematics. If a pupil expresses a rule or definition, first in language he understands, and then turns to the textbook for comparison with an accepted statement, it seems quite likely that any strange word in the textbook statement will be better understood.

It seems pertinent finally at this point to raise the question of what the relationship is between the discovery method of teaching mathematics and deductive proof. I believe that most current practice delays much attention to deductive proof until the time when demonstrative geometry is taught. Prior to that time, ideas, operations, and relationships are discovered inductively in the manner earlier described. A teacher then proceeds to use these ideas and operations without much attention to deduction. However, the teacher of general mathematics in the early years of high school will help the pupil for his study of deductive proofs later if she will regularly point out the limitations of conclusions reached inductively.

Edited by H. Glenn Ayre, Western Illinois State College, Macomb, Illinois

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Your professional dates

The information below gives the date, name, and place of meeting, with the name and address of the person to whom you may write for further information. For information about other meetings, see the previous issues of THE MATHEMATICS TEACHER. Announcements for this column should be sent at least ten weeks early to the Executive Secretary, National Council of Teachers of Mathematics, 1201 Sixteenth Street N.W., Washington 6, D. C.

NCTM convention dates

April 13–16, 1955
ANNUAL MEETING
Statler Hotel, Boston, Massachusetts
Jackson Adkins, local chairman, Phillips Exeter Academy, New Hampshire

July 4, 1955
JOINT MEETING WITH NEA
Chicago, Illinois
E. H. C. Hildebrandt, local chairman, Northwestern University, Evanston, Illinois

August 21–24, 1955
SUMMER MEETING
Indiana University, Bloomington, Indiana
Philip Peak, local chairman, Indiana University, Bloomington, Indiana

December 27–30, 1955
CHRISTMAS MEETING
Sheraton-Park Hotel, Washington, D. C.
Veryl Schult, local chairman, Wilson Teachers College, Washington 9, D. C.

Other professional dates

January 15, 1955
Mathematics Conference
University of Michigan, Ann Arbor, Michigan
Phillip S. Jones, Department of Mathematics, University of Michigan

March 4–5, 1955
Mathematics Conference of the Department of Mathematics of the North Carolina Education Association
University of North Carolina, Chapel Hill, North Carolina
Annie John Williams, 2021 Sprunt Street, Durham, North Carolina

April 13–14, 1955
Mathematics Conference of the Ontario Association of Teachers of Mathematics and Physics
University of Toronto, Toronto, Ontario
Ern Totton, Forest Hill Collegiate Institute, Toronto, Ontario

Troubles with zero

"Query 16:
Whether certain maxims do not pass current among analysts which are shocking to good sense? And Whether the common assumption, that a finite quantity divided by nothing is infinite, be not of this number?"—*Bishop Berkeley in the Analyst (1734)*

Note on above quotation: According to Florian Cajori (THE MATHEMATICS TEACHER, Volume 22, p. 366), Martin Ohm, the brother of George Ohm of "Ohm's Law" fame, is the first writer to define the operation of division in algebra and the first to give the proof (in 1828) that division by zero is not possible.

Reviews and evaluations

Edited by Richard D. Crumley, University of South Carolina, Columbia, South Carolina, and Roderick C. McLennan, Arlington Heights Township High School, Arlington Heights, Illinois

BOOKS

Basic Techniques of Mathematics, Howard S. Kaltenborn, Samuel A. Anderson, Helen H. Kaltenborn, Memphis State College, Memphis, Tennessee, 1954. Paper, 167 pp., $2.75.

This book has a minor revision in content from the average intermediate algebra text generally used in high school third-semester algebra. The change that has been made in content is the inclusion of trigonometric functions and the solution of triangles. The presentation of the subject matter is straightforward. The illustrative problems are well chosen and simple enough for the student to read.—*Orlando C. Kreider, Iowa State College, Ames, Iowa.*

Henri Poincaré—Critic of Crisis—Reflections on His Universe of Discourse, Tobias Dantzig, New York, Charles Scribner's Sons, Twentieth Century Library, 1954. Cloth, xi+149 pp., $3.00.

One hundred years ago France was graced with the birth of a child who eventually became one of the intellectual giants of the modern world. Rare is the phenomenon among scientists and mathematicians of a personality that influenced many and diverse fields. This Mathematical Hall of Fame has more places to occupy than there are eligible occupants. Archimedes, Euler, Gauss, Newton, . . . and Poincaré.

Dr. Dantzig is one of the few fortunate ones who studied with Poincaré, and this fact makes him eminently qualified for writing an essay depicting the intellectual contributions of Poincaré, while keeping purely biographical topics in the background.

It is extremely difficult to compress everything concerning Poincaré in one hundred and fifty pages. Here is a Gallic mind *par excellence.* Here is a spiritual adventure, so light, almost elfic, while dealing with topics that would floor anyone. Here is a mathematician who had no fears of venturing far afield into the exact sciences and philosophy and who was successful in making major contributions. Here is a master of exposition who could convey the most difficult ideas so popularly that Poincaré might be acknowledged as the originator of the movement of making difficult things easy.

What other recommendation can one make to a teacher of mathematics? If you want to learn what the *moderns* have done in and for mathematics, get this book and *read* it. The rewards that one will reap from such an experience are immeasurably greater than the investment. —*Aaron Bakst, New York University, New York, N. Y.*

Mathematics for the Secondary School—Its Content, and Methods of Tecaching and Learning, William David Reeve, New York, Henry Holt and Company, 1954. Cloth, vii+547 pp., $5.95.

It is difficult to believe that anyone could read this text and not be impressed with the author's primary concern with the learning processes designed to make the teaching of secondary mathematics more effective. While the author served in the capacity of editor of THE MATHEMATICS TEACHER, he became intimately acquainted with those teaching problems which were of most concern to both secondary teachers and leaders in mathematics education. The text gives ample evidence of this experience and acquaintance. The publication is primarily written for the collegiate level and designed to orient both teachers and student teachers to the problems of classroom teaching and their suggested solutions. A functional approach is chosen by orientation to brief historical development of teaching problems, decidedly favoring an integrated organization of concepts. Such an organization is consistent with the author's previously expressed position, and evidence is presented to indicate why such an organization would be more effective in meeting both the needs of the scientist and those of the general educator.

"Questions and Topics for Discussion" are presented at the end of each chapter. The problems indicate care in selection for further study and motivation, but an associated bibliography would have been helpful for the more interested student. The author's conversational style tends to make extended use of the personal pronoun and of "thus," making some statements obvious over-generalizations. Such statements are largely limited to the author's own experience as a classroom teacher, but must be taken as one teacher's opinion and not confused with his reference to results of an investigation or recommendation of a recognized committee or commission

An excellent discussion of student teaching in mathematics is provided, as well as some of the basic issues confronting the preparation of

teachers of secondary school mathematics. This presentation alone could justify a place in each library concerned with teacher education in mathematics. Discussions of the teaching of informal geometry and demonstrative geometry seem more adequate than the teaching of algebra, although algebra is discussed about as adequately as one will find in most similar texts. There appears to be less consideration given to discussions of approximate computation, directed numbers, and solution of verbal problems, yet each is considered in association with teaching areas. At times the discussion of a teaching problem becomes too finite, such as "The Conduct of the Recitation" in the teaching of demonstrative geometry, and in the description of a mathematics classroom. Teachers may find such particularization helpful. The "nature of proof" is accepted as being a primary objective in the teaching of demonstrative geometry, but those teachers and students favoring a method of teaching geometry as described by Fawcett in the Thirteenth Yearbook may feel that their particular interest has not been sufficiently emphasized. The problems of the junior high school are excellently organized and sufficiently comprehensive.

Because of the great concern with the place of the present concept of "general mathematics" in the beginning years of high school, the chapter on "The History and Teaching of General Mathematics" should be very helpful. The author's forthright statements substantiated by investigations, commission recommendations, and own experience should do much in providing feasible suggestions for finding a more satisfactory solution than what presently characterizes the teaching of first-year secondary mathematics.

This text should become a part of those libraries concerned with mathematics education and the classroom libraries of those teachers concerned with innovations designed to improve the organization and teaching of mathematics. Dr. Reeve has met an expressed need by summarizing what he considers to be the best expression of suggestions for improving the teaching of secondary mathematics. Some readers may find insufficient treatment of particular topics, but conscientious teachers will find much to challenge their thinking on how to improve their teaching. This publication is a contribution to the literature of mathematical education, and will be welcomed by the author's many students and friends.—*Bjarne R. Ullsvik, Illinois State Normal University, Normal, Illinois.*

EQUIPMENT

Geometric Figures and Solids (#8064)
Milton Bradley Company, Springfield 2, Massachusetts.
Set of 17 cardboard plane figures and 22 hardwood solids, with $8'' \times 5\frac{1}{2}'' \times 4\frac{1}{4}''$ clear plastic box to store figures and solids; $11.00 per set.

The cardboard plane figures are small—all but one would fit into the palm of one's hand. The solids are smooth pieces of unfinished hardwood, with the longest dimension three inches or less. The solids include five different prisms, two cylinders, a cone, a sphere, a hemisphere, an ovoid, an ellipsoid, and an oblate spheroid.

This set can be used very effectively in instruction in the fields of intuitive geometry, solid geometry, and analytical geometry. The solids are too small to be seen well by a large class of students, but they could be passed around easily. The cardboard plane figures seem worthless—it would be far better for students to make such materials—but the solids will be found to be very useful. These solids may inspire some boys skilled in wood-working to make duplications of, or additions to, the set. The price may seem high, but the quality of the wood solids is excellent.

Rubber Fraction Pies (#N230)
Creative Playthings, Inc., 5 University Place, New York 3, New York.
Set of eight 7" diameter rubber pies ($\frac{3}{8}''$ thick); set of eight pies, $6.50.

The set consists of eight rubber pies, each of which has been cut into either halves, thirds, quarters, fifths, sixths, eighths, tenths, or twelfths. Each pie has a different color and is pliable yet sufficiently firm to hold its shape.

These materials were designed for use in the elementary grades, but could be used profitably in the seventh or eighth grade for review or remedial purposes. They could be used for developing understandings and skills involving equivalence of fractions and the four fundamental operations using fractions. These materials might also be useful in the area of intuitive geometry. The circumference of a pie could easily be measured using a tape measure and compared to the radius of the "circle"; or the area of a "circle" could be successively approximated using fourths, fifths, tenths, and twelfths arranged not as a pie but as a pseudo-rectangle.

The workmanship illustrated in these materials is good; however, the three pies cut into eighths, tenths, and twelfths in the set examined did not fit together as well as the other five pies. On the whole, these materials are durable and well constructed.

There are many producers of instructional materials, so you can help to make this department useful to other teachers by informing the editors about new materials or old materials not widely used. In addition to this, you are encouraged to send comments and criticisms of the evaluations given in this department. If you disagree with an evaluation that has been published, send us your own evaluation of the material involved for possible later publication. Our goal is to serve you.

• WHAT IS GOING ON IN YOUR SCHOOL?

Edited by John A. Brown, University of Wisconsin, Madison 6, Wisconsin, and Houston T. Karnes, Louisiana State University, Baton Rouge 3, Louisiana

Science fair information

Contributed by Dewey E. Large, Oak Ridge Institute of Nuclear Studies, Oak Ridge, Tennessee

The Oak Ridge Institute of Nuclear Studies is attempting to meet the great need for increased interest in and knowledge of science and mathematics through a program of activity designed and spearheaded by its Field Representative, Dewey E. Large, a former science and mathematics teacher and school administrator. For several years the Institute has served as an academic liaison for disseminating education directly related to nuclear fission and fusion. This new educational endeavor is concerned with all branches of the pure physical and biological sciences in all their related and applied aspects. A wholesome stimulation of interest in acquiring purposeful knowledge of science and mathematics and opportunities for realizing and demonstrating potential or kinetic talent in these fields is absolutely necessary in order that adequate numbers of scientists may be continuously made available. Especially in the thirteen southeastern states and Puerto Rico, where this Oak Ridge program is being established, institutions of higher education, industry, agriculture, research foundations, and government are feeling the vital need of more local men and women who have education and training in science.

Scientists should not have to be imported to this area. A problem exists which must be solved. The need must be filled if competition is to be met and status improved. Primarily the problem is one of education and basically the need can be met only if elementary and secondary school teachers are given opportunity, recognition, and compensation.

The Field Representative has the sanction of scientists and educators in his approach and in making known to the public that the Oak Ridge Institute of Nuclear Studies offers its facilities in promoting and advancing science. Currently, concentration is upon science fairs as the means of getting these results. A science fair will be considered as a collection of exhibits, each of which is designed to show a biological, chemical, physical, or engineering principle; a laboratory or other procedure; an industrial development; or an educational and orderly collection which can be taken as fitting into the concept of any branch of applied science. These exhibits should be designed and made by students, with interested teachers providing inspiration and guidance.

This year, the Field Representative's mission is to assist in establishing and improving science fairs. The first step will be the setting up and holding of well-organized science fair work conferences strategically located throughout the southeastern area of the United States. The science fair work conferences are designed and proposed to stimulate interest in matters pertaining to organization, administration, and coordination of science fairs with the expectation of starting a chain reaction of activity in the advancement of science and mathematics.

Three of these work conferences have been arranged in cooperation with state universities and state academies of science, one to be held at the University of North Carolina, Chapel Hill, October 15 and 16, for the states of North Carolina, South Carolina, and Virginia; a second to be held at the University of Georgia, Athens, November 5 and 6, for the areas of Georgia, East Tennessee, East Alabama, Florida, and Western South Carolina; and a third at the University of Mississippi, Oxford, November 12 and 13, for the states of West Alabama, Middle and West Tennessee, Kentucky, Arkansas, Louisiana, and Mississippi. Representatives from industry, communication media, state departments of education, and representatives from institutions of higher education are invited to attend, along with educational administrators, instructional supervisors, and science and mathematics teachers from public, private, and parochial schools.

A scaling device for grading papers

Contributed by Joseph Kennedy, University of Wisconsin High School, Madison, Wisconsin

It is frequently necessary to convert a set of raw scores to numerical grades. When the score range equals the grade range this is easily accomplished by adding a constant to each score. But when the ranges are not equal, the scaling is not so easy and doubtful approximations are usually made. The following method, which is rapid and easy, gives results that are as precise and accurate as the raw scores.

Let

y represent the raw score

x represent the amount to be added to a score to make it a grade

y_2 be the highest score

x_2 the amount to be added to y_2 to make it the highest grade

y_1 the lowest passing score

x_1 the amount to be added to y_1 to make it the lowest passing grade

substitute these values in the straight line equation $(y-y_1)(x_2-x_1)=(x-x_1)(y_2-y_1)$

simplify to the form $ky+c=x$

add y to both sides giving $y(k+1)+c=x+y=\text{grade (numerical)}$.

On a slide rule set a C index over $k+1$ on the D scale, read $y(k+1)$ by inspection and add c mentally to get grades.

An example:

change these scores

30–39	1
40–49	1
50–59	6
60–69	2
70–79	12
80–89	9
90–99	3

to grades in the range:

70 (lowest passing)

↓

100 (highest)

$y_1=52$

$x_1=18$

$y_2=96$

$x_2=4$

Substituting in the straight line equation and simplifying gives:

$$-.32y+34.6=x$$

add y, $.68y+35=x+y$ or grade. Set a C index over .68 on the D scale, $.68y$ may then be read by inspection and 35 added mentally to get grades. Points other than lowest passing and highest may be used to set k and c.

Edited by Paul C. Clifford, State Teachers College, Montclair, New Jersey, and Adrian Struyk, Clifton High School, Clifton, New Jersey

Miscellaneous bibliographical notes

by Adrian Struyk

Trigonometry without Tables

Cheney, W. F. "Rational Approximations for Trigonometric Functions." *National Mathematics Magazine* (April 1945), v. 19, pp. 341–342.

This paper develops the function

$$f(x)=(x/7)(22-x^2)/(4-x^2)$$

and shows how it can be used for approximating (to five decimal places) the trigonometric functions of any acute angle. Let Y be such an angle. Take $v=\frac{1}{2}Y$. Then we have

$$\sin Y=(2\tan v)/(1+\tan^2 v),$$
$$\cos Y=(1-\tan^2 v)/(1+\tan^2 v),$$
$$\tan Y=(2\tan v)/(1-\tan^2 v).$$

That is, any trigonometric function of Y is a rational function of $\tan v$. By taking $x=v/45$, we get $f(x)=\tan v$ very nearly. Corrections are given.

Frame, J. S. "Solving a Right Triangle Without Tables." *American Mathematical Monthly* (Dec. 1943), v. 50, pp. 622–623.

It is shown that if A is the smallest angle of right triangle ABC ($C=90°$) then the number of degrees in angle A is given very nearly by $A=172a/(b+2c)$. The correction in minutes to be added to the computed value A is equal to nearly

$$(-1.2)(A/30)+(.8)(A/30)^5.$$

Stelson, H. E. "Approximate Solution of an Oblique Triangle Without Tables." *American Mathematical Monthly* (Feb. 1949), v. 56, pp. 94–95.

Here we find that in an oblique triangle ABC the radian measure of an acute angle A is very nearly equal to

$$\frac{6\sqrt{(s-b)(s-c)}}{2\sqrt{bc}+\sqrt{s(s-a)}},$$

the error amounting to about $.00035A^5$ radians. Analogous expressions approximate the angles B and C if the triangle is acute. If one angle (say C) is obtuse a better approximation is obtained by computing the *supplement* of C. This is very nearly equal to

$$\frac{6\sqrt{s(s-c)}}{2\sqrt{ab}+\sqrt{(s-a)(s-b)}}.$$

The approximations by Frame and by Stelson are based on the expansion

$$\frac{2}{\sin x}+\frac{1}{\tan x}=\frac{3}{x}+\frac{x^3}{60}\cdots,$$

easily derived from the power series for $\csc x$ and $\cot x$, and also readily verified numerically.

Crossed Ladders in an Alley

Here we are concerned with a "perennial" problem, the essential features of which are as follows:

In a plane, AB and CD are both perpendicular to BD, and on the same side of BD. AD and CB intersect at P. M is the foot of the perpendicular from P to BD. If AD, CB, PM are given lengths a, b, c, respectively, find the length d of BD.

Let $AB=x$, $CD=y$. Then the main property of the figure is expressed in the

equation $1/x+1/y=1/c$, or $c=xy/(x+y)$. For this to be true AB, CD, PM must be parallel to each other, but they need not be perpendicular to BD. With right angles, however, as specified above, the Pythagorean Theorem yields the equation

$$(a^2-d^2)^{-1/2}+(b^2-d^2)^{-1/2}=c^{-1}.$$

Clearing of fractions and radicals results in an equation of degree four in d^2. Straightforward algebraic solution of this equation would not be considered a "recreation" by most of us. Various artifices have been used in attempts to effect a neat solution of the problem. Interesting aspects can be found in the references given below.

American Mathematical Monthly:

V. 43, pp. 642–643, Dec. 1936. Problem E210. Determines the set of integers which involve the "smallest integral length of the longest ladder." $a=105$, $b=87$, $c=35$, $d=63$.

V. 48, pp. 268–269, April 1941. Problem E433. Exhibits a four parameter solution for determining sets of integers.

National Mathematics Magazine, v. 19, pp. 205–207, Jan. 1945. Problem #567. A general solution by a trigonometric method.

YATES, R. C. "The Ladder Problem." *School Science and Mathematics*, v. 51, pp. 400–401, May 1951. A graphical treatment.

ANNING, N. "New Slants on Old Problems." THE MATHEMATICS TEACHER, v. 45, pp. 474–475, Oct. 1952. A trigonometric slant.

Specific numerical cases solved by various methods are as follows: *The Pentagon*, v. 10, p. 98, problem 32. Given $a=40$, $b=30$, $c=10$; found $d=26.04$.

School Science and Mathematics:

V. 32, p. 212, Feb. 1932. #1194. Given $a=100$, $b=80$, $c=10$; found $d=79.10$.

V. 37, pp. 860–861, Oct. 1937. #1498. Given $a=40$, $b=30$, $c=15$; found $d=15.99$.

V. 49, pp. 244–245, Mar. 1949. #2116. Given $a=60$, $b=40$, $c=15$; found $d=33.75$. Consult this solution for additional references.

A note on cologarithms

by Sam Selby, University of Akron, Akron, Ohio

Recent publications of elementary texts in algebra and trigonometry stress the importance of scientific notation for numbers as it relates to the convenience of finding the characteristics for the common logarithms of numbers. This follows from the fact that if $N=P\cdot 10^k$, where $1\leqq P<10$, $\log N=\log P+k \log 10=\log P+k$, and of course the characteristic is k, the $\log P$ being in toto the mantissa, and contained directly in regular common logarithmic tables.

The writer of this note wishes to point out an analogous situation exists for characteristics for the common cologarithms of numbers. This follows from the fact that if $N=R\cdot 10^m$, where $0.1\leqq R<1$, then $\text{colog } N=\text{colog } R+m \text{ colog } 10=\text{colog } R-m$, and $-m$ is the characteristic for colog N. For example, the characteristics for colog 0.000015 and colog 57.2 would respectively be 4, and -2, since $0.000015=0.15\cdot 10^{-4}$ and $57.2=0.572\cdot 10^2$.

If a table of common logarithms is available which gives directly the negative decimal values for the range included for R, it becomes apparently simple to find the common cologarithm of any particular number. Thus if log 0.15 and log 0.572 were contained directly in a table as -0.8239 and -0.2434, it would follow that colog $0.000015=4.8239$ and colog $57.2=0.2434-2=8.2434-10$.

It may be noted that such a four-place common logarithm table of the type mentioned herein is to be found either in the Mathematical Tables from the *Handbook of Chemistry and Physics*, pages 16 and 17, or its recent successor, the tenth edition of the *Standard Mathematical Tables*, pages 18 and 19.

When teaching cologarithms of numbers, the experience of the writer has been that it is easier to present and easier for the students to understand this topic when using this rule and this table.

For your guidance folder

The quotation below was taken from the *Newsletter* (#65, September 7, 1954), published jointly by the Engineering Manpower Commission of Engineer's Joint Council and the Scientific Manpower Commission. It is excellent material to put in a guidance folder to have on hand for those occasions when you discuss the opportunities for women in science and mathematics.

WOMEN IN SCIENCE

"Representatives of industry and education met at Bryn Mawr recently to consider the recruitment of women for careers in science. The summary of conclusions merits careful study, and, though basically sound, it suggests that some important factors in career selection may not have been given as much weight as they deserve. Some of these follow. Chief among them are the domestic and social environment in which most young women are raised and the lingering reluctance of industry to employ women for technological and scientific work.

"The record of earned degrees in 1953 shows that mathematics and biology attract women students in greatest numbers, whereas engineering draws the fewest:

	Bachelor's Degrees			Master's Degrees			Doctor's Degrees		
	total	women	%	total	women	%	total	women	%
Mathematics	4,396	1,274	29	677	112	16	241	14	6
Biology	9,707	2,745	28	1,891	346	18	966	117	12
Chemistry	5,943	1,073	19	1,211	116	10	999	51	5
Geology	1,719	68	4	517	8	2	133	3	2
Physics	102	1	1	50	0	—	13	0	—
Engineering	24,189	37	0.15	3,566	13	0.03	518	1	0.02

"In the total college population women comprise roughly one third, and the percentage drops but slightly through the master's degree level in the graduate schools. At the doctorate level, however, fewer than 10% of the degrees granted go to the women. Among the sciences only mathematics and biology approach the norm in the undergraduate field, but both follow the same curve of rapid decline in the graduate schools. As compared with the record in the social sciences and humanities, the figures reveal that a small proportion of women take advanced training for the master's degree, and that, except in biology, the number who take the Ph.D. is negligible.

"For the many women who are seriously interested in science careers there are severe handicaps. Women as a class are more eminently suited for certain types of scientific work than for others, and in some, experience shows that they excel men. Yet there is no attempt to give academic recognition to the mental and manual differences between men and women, and the latter not infrequently resent any implication that they exist. In job opportunities—and sometimes in the classroom—women find themselves at a competitive disadvantage with men, and most of those who stay in the profession know that they have to be well above the male average in mentality and skill to succeed. And for the most part the professional woman is still so rare as to be a social curiosity, and the choice of a career in science consigns her to social isolation.

"These and other qualifying factors must be recognized before womanpower will add substantially to our scientific and technological strength. Efforts should be made to remove whatever industrial prejudices and social hurdles still bar the way to science careers for women and to plan more judicious university curricula. Meanwhile, emphasis may well be placed on the service they can perform as teachers of science. Of the 4,000 students who completed high school teaching certificate requirements in science in 1953, only 1,150 were women, although two thirds of them actually accepted teaching posts as contrasted with 43% of the 2,825 men. It is not out of order to urge college and university departments to turn their attention to the instructional needs of prospective high school teachers of both sexes, and to introduce courses that will attract, as well as train, new teachers."